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Subject : Differential equation

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Q#01 (a): Estimate the general solution of $y' = (x+2)y^2$.

Solution:

$$y = (x+2)y^2$$

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

$$\Rightarrow \int \frac{1}{y^2}\, dy = \int (x+2)\, dx$$

$$\Rightarrow \int y^{-2}\, dy = \int (x+2)\, dx$$

$$\Rightarrow \frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\Rightarrow \frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

Multiplying both sides by -1

$$\Rightarrow y^{-1} = -\left(\frac{x^2}{2} + 2x + C\right)$$

$$\Rightarrow y = -\left(\frac{1}{x^2/2 + 2x + C}\right)$$

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Q#01 (b): Estimate the general Solution of $y' = (y+9x)^2$ ..(i)

Solution: Let $y + 9x = u$.

$$\Rightarrow \frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 9$$

So (i) become

$$\Rightarrow \frac{du}{dx} - 9 = u^2$$

$$\Rightarrow \frac{du}{dx} = u^2 + 9$$

$$\Rightarrow \int \frac{1}{u^2+9}\, du = \int dx$$

$$\Rightarrow \int \frac{1}{(3)^2+u^2}\, du = \int dx$$

$$\Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) = x + C_1$$

$$\Rightarrow \tan^{-1}\left(\frac{u}{3}\right) = 3x + 3C_1$$

$$\Rightarrow \frac{u}{3} = \tan(3x + C)$$

$$\Rightarrow u = 3\tan(3x + C)$$

$$\Rightarrow y + 9x = 3\tan(3x + C)$$

$$\Rightarrow y = -9x + 3\tan(3x + C)$$

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Q#02 (a): Estimate the general Solution of

$$x^3 dx + y^3 dy = 0$$

Solution: $x^3 dx + y^3 dy = 0$.

$\Rightarrow M du + N dy = 0$.

$\Rightarrow M = x^3$, $N = y^3$.

$$\Rightarrow \frac{\partial M}{\partial y} = 0 \cdot \frac{\partial N}{\partial x} = 0.$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ so exact.

$$\Rightarrow u = \int M dx + K(y).$$

$$\Rightarrow u = \int x^3 du + K(y).$$

$$\Rightarrow u = \frac{x^4}{4} + K(y) \quad -(i)$$

$$\Rightarrow \frac{\partial u}{\partial y} = 0 + \frac{d}{dy} K(y)$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{d}{dy} K(y)$$

We know that

$$\Rightarrow \frac{\partial u}{\partial y} = N = y^3$$

$$\Rightarrow y^3 = \frac{d}{dy} K(y) \Rightarrow \int y^3 = \int K(y)$$

$$\Rightarrow K(y) = y^4/4 + C_1 \text{ . Putt in (i)}$$

$$\Rightarrow C_1 = \frac{x^4}{4} + \frac{y^4}{4} + C_2$$

$$\Rightarrow C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\Rightarrow C_2 - C_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\Rightarrow C = \frac{x^4}{4} + \frac{y^4}{4} \quad \text{Answer.}$$

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Q #:03 (b): Estimate general Solution.

$$4y'' - 6y' - 7y = 0$$

Solution: Assume $y(x) = e^{\lambda x}$

put in eq

$$\Rightarrow 4 \cdot \frac{d^2 \cdot y(x)}{dx^2} - 6\frac{d}{dx} y(x) - 7y(x) = 0$$

$$\Rightarrow 4 \cdot \frac{d^2}{dx^2}(e^{\lambda x}) - 6\frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0 \quad \text{eq(i)}$$

$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \rightarrow (A)$$

$$\Rightarrow \frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \rightarrow (B)$$

put (A) and (B) in eq(i)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow (4\lambda^2 - 6\lambda - 7)e^{\lambda x} = 0$$

$$\Rightarrow \lambda = 3/4 - \sqrt{37}/4$$

$$\Rightarrow \lambda = 3/4 + \sqrt{37}/4$$

$$\Rightarrow y(x) = y_1(x) + y_2(x)$$

$$\Rightarrow y(x) = c_1 e^{(3/4 - \sqrt{37}/4)x} + c_2 e^{(3/4 + \sqrt{37}/4)x}$$

Q #03 (a) : Find the general solution

$$4y'' - 20y' + 25y = 0$$

Solution: This is second order homogenous differential equation with constant coefficient

$$ay'' + by' + cy = 0$$

and the solution for this is $y = e^{\lambda}$ —(i)

General solution:

$$y = C_1 e^{\lambda} + C_2 + e^{\lambda}$$

now

$$4\frac{d^2}{dx^2}(y) - 20\frac{d}{dx}(y) + 25(y) = 0$$

eq (A)

put eq(i) in eq(A)

$$\Rightarrow 4\frac{d^2}{dx^2}(e^{\lambda x}) - 20\frac{d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0$$

$$\Rightarrow \frac{d^2}{dx^2}e^{\lambda x} = \lambda^2 e^{\lambda x} \longrightarrow (B)$$

put eq(B) and eq(i) in eq(A)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 20\, e^{\lambda x} + 25\, e^{\lambda x} = 0$$

$$\Rightarrow e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$$

$$\Rightarrow e^{\lambda x} \neq 0$$

$$\Rightarrow 4x^2 - 20\lambda + 25 = 0$$

$$\Rightarrow (2\lambda - 5)^2 = 0$$

$$\lambda = 5/2 \quad \text{or} \quad \lambda = 5/2$$

$$\Rightarrow y(x) = y_1(x) + y_2(x)$$

$$\Rightarrow \boxed{y(x) = C_1 e^{5/2\,x} + C_2 x e^{5/2\,x}}$$

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