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Sec : A

Subject: Differential Equation

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Sub. To

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Q No 1

$$\frac{dy}{d\cancel{x}t} = e^{y-t} \sec(y)(1+t^2) \qquad y(0)=0$$

Sol :-

$$\frac{dy}{dx} = e^{y-t} \sec(y)(1+t^2)$$

$$y(0) = 0 \qquad \text{So} \quad x = 0 \qquad y = 0$$

$$dy = e^{y} \cdot e^{-t} \sec(y)\,(1+t^2)\, \cancel{dx}\, dt$$

$$\frac{1}{e^{y} \cdot \sec(y)} dy = (1+t^2) e^{-t}\, \cancel{dx}\, dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y} \cos y \, dy = \int (1+t^2)\, e^{-t}\, \cancel{dx}\, dt$$

using integration by parts

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) =$$

$$(1+t^2) \int e^{-t} \cdot \frac{d}{dt} (1+t^2) \rightarrow \text{eq } 1$$

L.H.S

$\rightarrow$ $e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy} e^{-y}\right)$

$\rightarrow$ $e^{-y}\sin y - \int\left(\sin y \cdot e^{-y}(-1)\right)$

$\rightarrow$ $e^{-y}\sin y + \int\left(\sin y \cdot e^{-y}\right)$

$\rightarrow$ $e^{-y}\sin y + \int\left(e^{-y}\sin y\right)$

Again using integration by parts

$\rightarrow$ $e^{-y}\sin y + e^{-y}(-\cos y) - \int\left(\int \sin y \; ^{d}/_{dy}\, e^{-y}\right)$

$\rightarrow$ $e^{-y}\sin y + e^{-y}(-\cos y) - \int\left(-\cos y\,\frac{e^{-y}}{-1}\right)$

$\rightarrow$ $e^{-y}\sin y - e^{-y}\cos y - \int\left(\cos y\, e^{-y}\right)$

$\rightarrow$ Since $\int\left(\cos y\, e^{-y}\right) = LHS$

Since it is same again to the first one so L.H.S will become.

$\rightarrow$ $LHS = e^{-y}(\sin y - \cos y) - L.H.S$

$\rightarrow$ $2LHS = e^{-y}(\sin y - \cos y)$

$\rightarrow$ $LHS = e^{-y}\left(\frac{\sin y - \cos y}{2}\right)$

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Now taking R.H.S

$\rightarrow \int (1+t^2)\, e^{-1} dt$

$\rightarrow (1+t^2) \int e^{-t} - \int \left(\int e^{-t} \frac{d}{dt} (1+t^2) \right)$

$\rightarrow -(1+t^2)\, e^{-t} - \int (-e^{-t} (2t))$

$\rightarrow -(1+t^2)\, e^{-t} + \int ((2t)\, e^{-t})$

Again using integration by Parts.

$\rightarrow -(1+t^2)\, e^{-t} + \left(2t \int e^{-t} - \int \left(\int e^{-t} \frac{d}{dt} 2t \right) \right)$

$\rightarrow -(1+t^2)\, e^{-t} + (-2t\, e^{-t} - \int (-e^{-t} 2)$

$\rightarrow -(1+t^2)\, e^{-t} + (-2t\, e^{-t} + \int (2e^{-t}))$

$\rightarrow -(1+t^2)\, e^{-t} + (-2te^{-t} - 2e^{-t}) + c$

$\rightarrow -(1+t^2)\, e^{-t} + -2te^{-t} 2e^{-t} + c$

$\rightarrow -e^{-t} - e^{-t} t^2 - 2t\, e^{-t} - 2e^{-t} + c$

$\rightarrow -(t^2 + 2t + 3)e^{-t} + c = R.H.S$

→ Now take L.H.S = R.H.S

→ $e^{-y}\dfrac{(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t}+c$

→ we know that

→ $t=0 \quad y=0$

put it above

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

→ $c = 5/2$

Put value of c

$\Rightarrow \dfrac{e^{-y}}{2}(\sin y - \cos y) = -(x^2+2+3)\, e^{-t}\, 5/2$

Ans

Q NO 2

$$(\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y} - \sqrt{x-y})dy = 0$$

Sol:

$$\rightarrow (\sqrt{x+y} + \sqrt{x-y})\, dx - (\sqrt{x+y} - \sqrt{x-y})\, dy = 0$$

$$\rightarrow \frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \longrightarrow ①$$

This is homogeneous differential eq in x and y to solve this put

$$y = vx$$

$$\rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

This eq ① beome.

$$\rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$\rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

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$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\cancel{v} + 1-\cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

Taking Integrals on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$\rightarrow \quad \frac{1+\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$$

$$\rightarrow \quad x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$\rightarrow \quad x+\sqrt{x^2-y^2} = c_1 \qquad \because \ 1/c = c_1$$

which is a required Solution.

Q No 3

$$(D^4 + D^2)\, y = 3x^2 + 4\sin x - 2\cos x$$

Sol:

$$(D^4 + D^2)\, y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow \quad f(D)\, y = f(x)$$

As it is non homogenous linear eq so solution will be

$$y = y_c + y_p \quad \text{--- (i)}$$

Complementry Solution y_c

$$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$$

Either $D^2 = 0 \Rightarrow D = 0$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow D = i$$

or $D = \boxed{0+i}$

Roots are real and complex

$\Rightarrow$ $y_c = C_1 e^{0x} + e^{0x} (C_2 \cos x + C_3 \sin x)$

$\Rightarrow$ $y_c = C_1 + C_2 \cos x + C_3 \sin x$

$\Rightarrow$ $y_p = \frac{1}{f(D)} F(x)$

$\Rightarrow$ $y_p = \frac{1}{D^4 + D^2} (3x^2 + 4\sin x - 2\cos x)$

$\Rightarrow$ $= \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$

$\Rightarrow$ $f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

again differentiating

$f''(D) = 12D + 2$

so for $D = 0$

$f''(0) = 12(0) + 2 = 2$

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So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow \quad y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} 2\cos x$$

putting $D=0$ in all

So $$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

So putting in eq $\rightarrow$ (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \frac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + \frac{3}{2} x^4$$