Name :- Hamza Khan Yousafzai

ID :- 7487

Subject :- Structure Analysis II

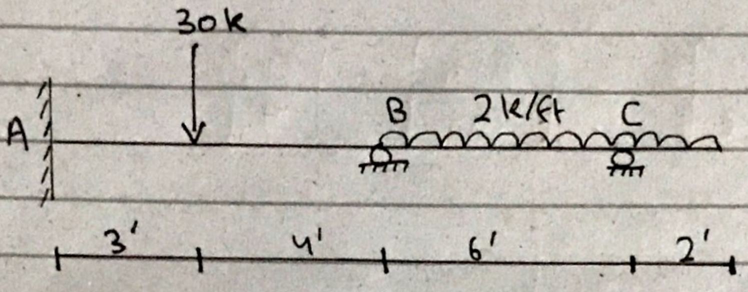
Semester :- 12th (Batch 2014)

Submitted to :- Engr. Adeed Khan

Final term

Qno 1 :-

Analyze the beam shown in FIG-1 by stifness method. Assume EI is constant



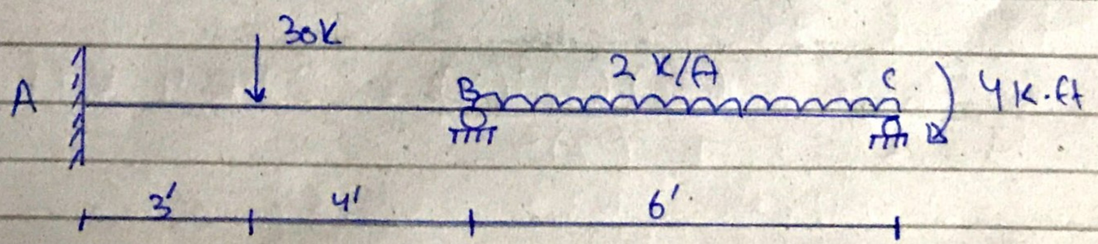
Solution:-

Step 1:-

Determining Kinematic Indeterminacy.

K.I = 5°

So we have to reduce the extended portion

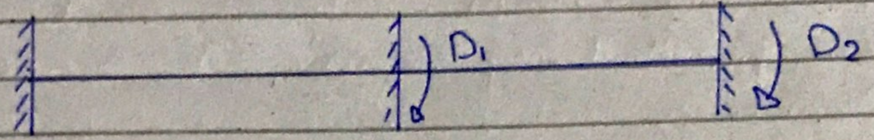


$\Rightarrow \frac{2(2)}{1} = 4$ K.ft

Now K.I = 2°

Step 2:-

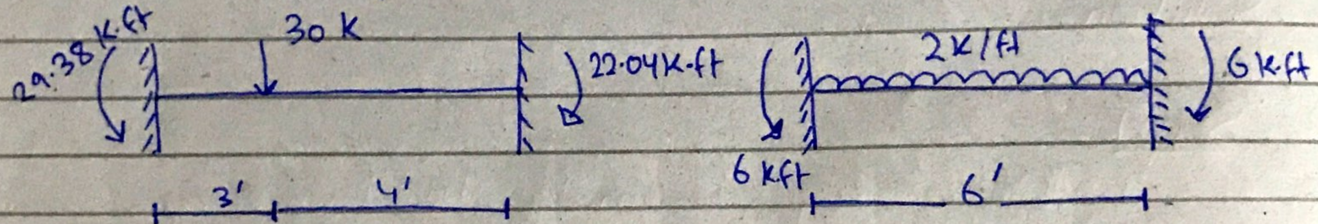
Determine Unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3:-

Compute [ADL] Matrix



⇒ For Pointed Load (Not at mid)

⇒ For Left end:-

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

⇒ For URL:-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k.ft}$$

$ADL_1 = 22.04 - 6 = 16.04$ k.ft

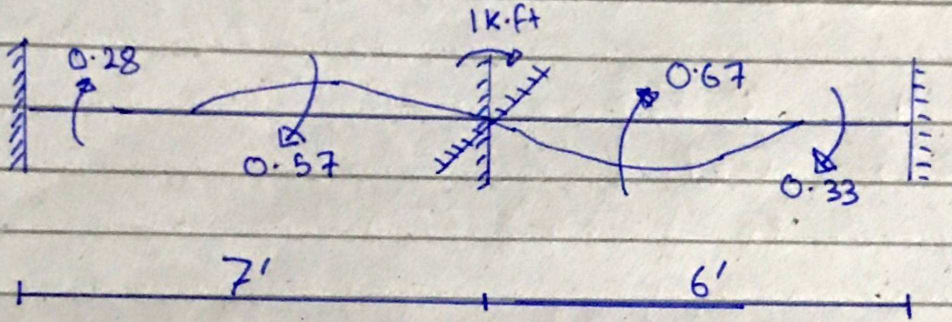
$ADL_2 = 6$ k.ft

Step 4 :-

Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a) $D_1 = 1k$, $D_2 = 0$



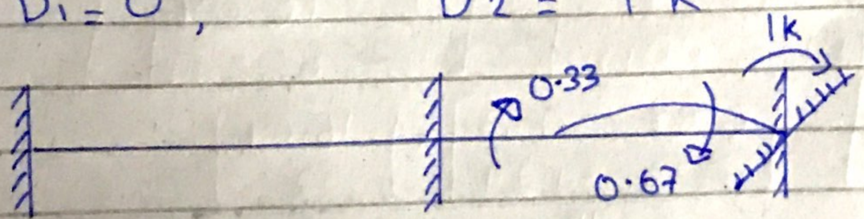
$\frac{4EI}{7} = \cancel{0.57}\ 0.57$ | $\frac{2EI}{6} = 0.33$

$\frac{4EI}{6} = 0.67$ | $\frac{2EI}{7} = 0.28$

$S_{11} = 0.57 + 0.67$
$= 1.24\ EA$

$S_{21} = 0.33\ EA$

(b) $D_1 = 0$, $D_2 = 1k$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$S_{12} = 0.33$$
$$S_{22} = 0.67$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step 5:-

Comput [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A \times \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} \cdot \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$
$$= 0.8308 - 0.1089$$
$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} \cdot 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

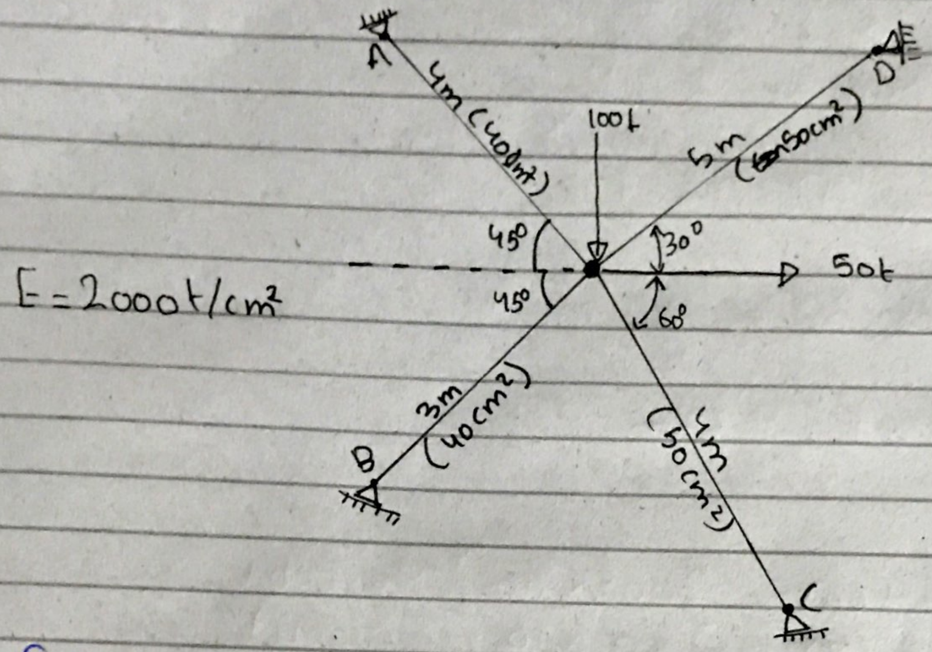
Now:- $\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & -1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.97 \\ 3.8902 \end{bmatrix}$$

Q no 2:-

Analyze the Pin-jointed frame shown by Stifness method. Length of members in 'm' and cross-sectional area of the members in cm^2 are shown in FIG. 3 Take $E = 2000\ t/cm^2$



⇒ Solution:-

⇒ For A:-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$\Rightarrow P = 2.828m$

$$\cos 45^\circ = \frac{b}{4}$$

$\Rightarrow b = 2.828m$

⇒ For B:-

$$\sin 45^\circ = \frac{P}{3}$$

$\Rightarrow P = 2.12\,m$

$\cos 45^\circ = \frac{b}{h}$

$\Rightarrow b = 2.12\,m$

$\Rightarrow$ For **D** :-

$\sin 30^\circ = \frac{P}{h = 5}$

$\Rightarrow P = 2.5$

$\cos 30^\circ = \frac{b}{5}$

$\Rightarrow b = 4.33\,m$

Now:-

$EA(A) = 2000 \times 40 = 80{,}000t$

$EA(B) = 2000 \times 40 = 80{,}000t$

$EA(C) = 2000 \times 50 = 100{,}000t$

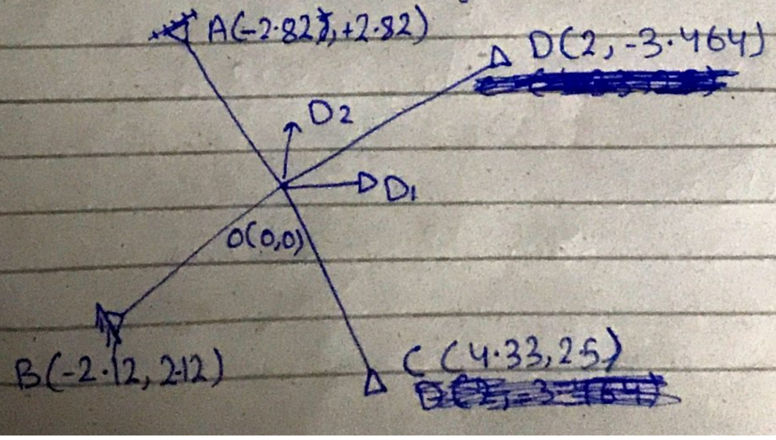
$EA(D) = 2000 \times 50 = 100{,}000t$

$\Rightarrow$ Step 1 :- K.I

$K.I = 2j - r$

$= 2(5) - 8 = 2^\circ$

Step 2 :- Select unknown joint displacement



$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$, $\begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$

Step 3:-

$[AMD]_{4\times 2}$ $\quad$ Σ, $[S]_{2\times 2}$

(i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80{,}000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100{,}000}{(500)^2} \times (0.43) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0-200) = -125$$

Now:-

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$$

$$= \frac{80{,}000}{400^3} \times (282)^2 + \frac{80000}{(300)^3} \times (212)^2 + \frac{10000}{(500)^3} \times (-433)^2 + \frac{100{,}000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

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$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_i)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

(iii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_i)$$

$$AMD_{12} = \frac{80{,}000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2}(346) = 216.25$$

Now:- $S_{22} = \sum_{j=1}^{m} \frac{EA}{L^3}(Y_K - Y_j)^2$

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$$\frac{80{,}000}{400^3}(-282)^2 + \frac{80{,}000}{300^3}(212)^2 + \frac{100000}{500^3}(-250)^2$$
$$+ \frac{100{,}000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step 4:-

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

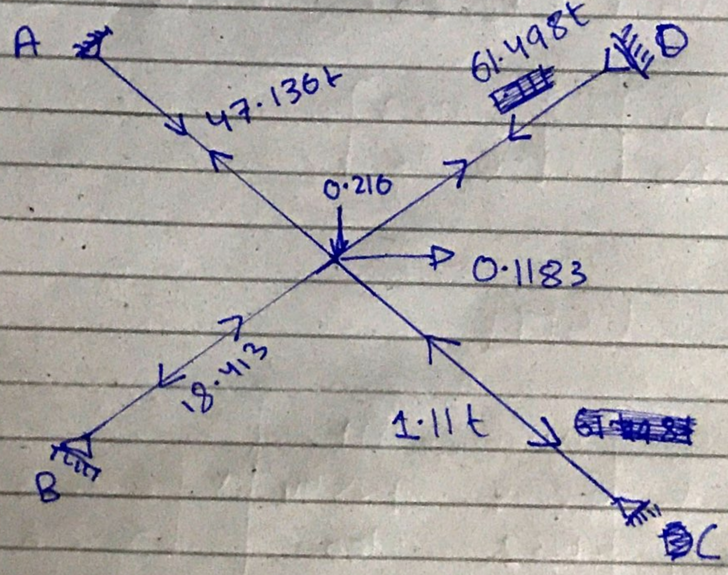
Step 6:- [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

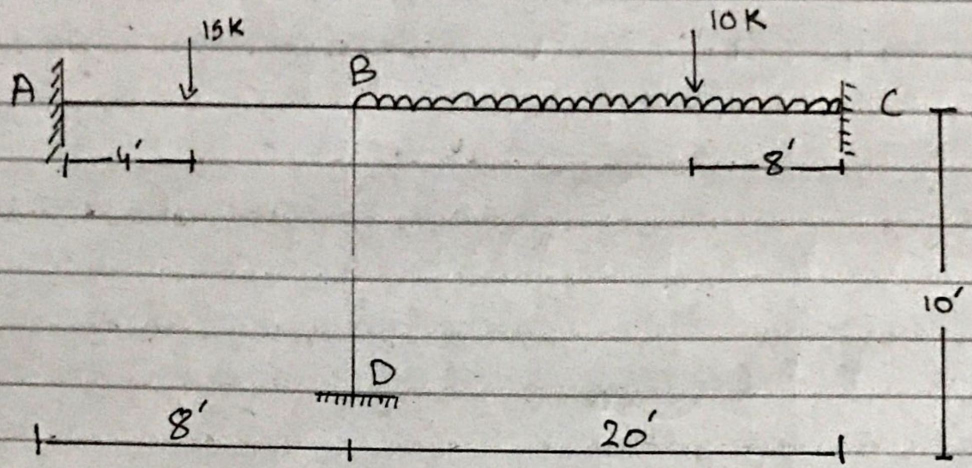
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 & +30.46 \\ 22.29 & -40.70 \\ -20.49 & +21.6 \\ -14.79 & -46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Q no 3:-

Analyze the rigid-joint frame shown in Fig 2 by stiffness method. Assume EI is constant



Solution :-

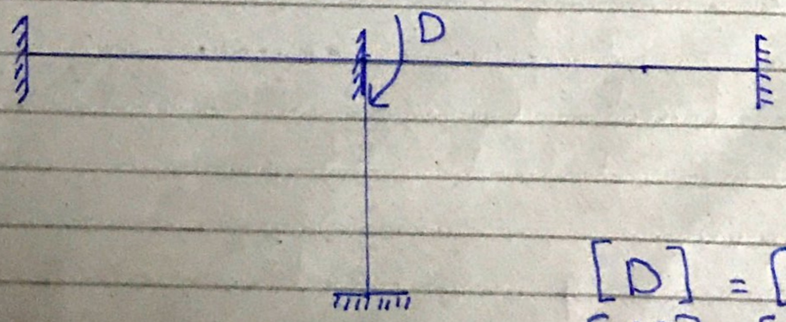
Step 1:-

Determine Kinematic Indeterminacy

K.I = 1°

Step 2:-

Determine Unknown Joint displacement

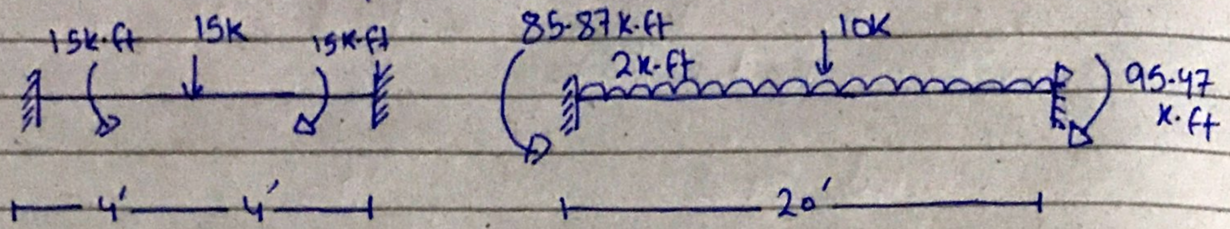


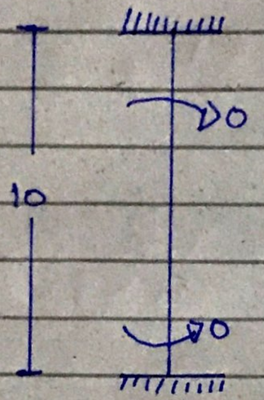
[D] = [?]

[AD] = [0]

Step 3:-

Compute [ADL] Matrix





⇒ Point load at Center :-

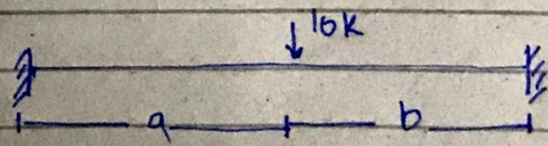
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip.ft}$$

⇒ Uniformly distributed Load:-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)}{12} = 66.67 \text{ k.ft}$$

⇒ Point Load (Not at mid) :-

Suppose



For Left End :-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ K.ft}$$

For right End :-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ K.ft}$$

So total moment at left End :-

$19.2 + 66.67 = 85.87$ K.ft

Similarly at right End :-

$28.8 + 66.67 = 95.47$ K.ft

So $[ADL] = -85.87 + 15 = -70.87$ K.ft

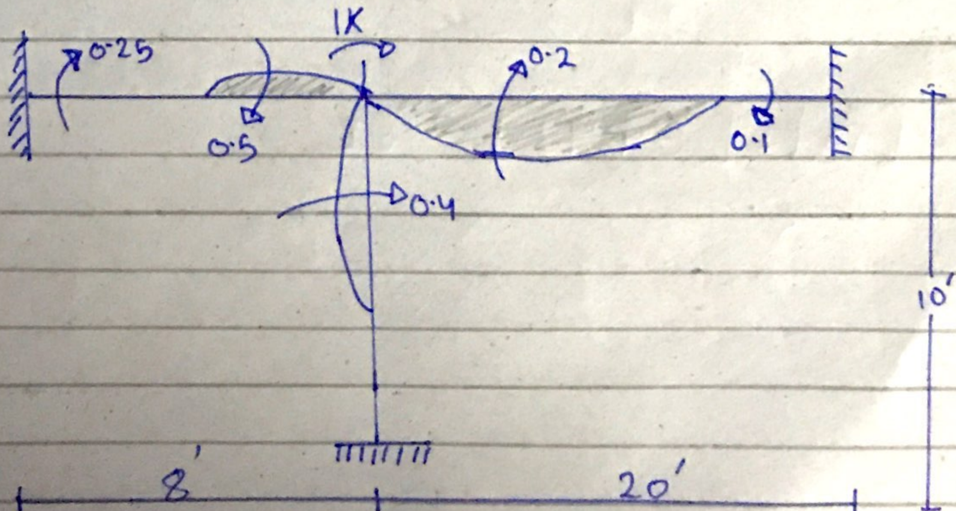
$\Rightarrow$ Step 4 :-

Determine [S] Matrix

$[S] = [S_{11}]$

Now

D = 1K



$\Rightarrow \frac{4EI}{8} = 0.5$ $\quad\quad$ $\frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2$ $\quad\quad$ $\frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4$ $\quad\quad$ $\frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)EI$

$= 1.1EI$

$[S] = 1.1EI$

$\Rightarrow$ Step 5 :-

Compute [D] Matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42] \; 1/EI$