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ID=7693

PAPER=EARTH QUAKE

EXAME=FINAL TERM SPRING 2020

SECTION=C

Question (1)

Given

$E = 29000 ksi$

$I = 150 in^4$

$\delta_{st} = 7693$

Solution

The general E.O.M for S.DOF system is $Ku + c\dot{u} + m\ddot{u} = P(t) \rightarrow$ (i)

⇒ In our case system is Undamped i.e ($c=0$)

⇒ Undergoing free vibration [$P(t)=0$]

⇒ Hence general EOM becomes

$$Ku + m\ddot{u} = 0 \longrightarrow \text{(ii)}$$

$K = 3EI / L^3$

Put value

$$= \frac{3 \times 29000 ksi \times 150 in^4}{10 \times 12 in^3}$$

But ⇒ $K = 7.55$ k/inch

To eliminate mistake during calculation use fundamental units like lb, ft, sec etc

So

$$K = 7.55\ \text{k/in} = \boxed{90625\ \text{lb/ft}}$$

Now

$$m = \frac{7693\ \text{lb sec}^2}{32.2\ \text{ft}} = 238.9$$

$$\boxed{m = 238.9\ \text{slug}}$$

Where as ⇒ $W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{96025}{238.9}}$

$$\boxed{W_n = 20.02}\ \text{rad/sec}$$

Now ⇒ $T_n = 2\pi / W_n = 2\pi / 20.02$

$$\Rightarrow \boxed{T_n = 0.313\ \text{sec}}$$

(3)

Now subtitute the coresponding value in eq (ii)

$\Rightarrow 90625 + 238.9u = 0$

But 'K' is in lb/ft and m is in lb sec/ft^2

So general solution for EOM for undamped free vibration is $\Rightarrow$ $u(t) = u(0)\cos(w_n t) \rightarrow$ (a)

$+ \dot{u}(0)/w_n \sin w_n t$

As $u(0) = \frac{1}{2} = \frac{1}{24\,ft}$ and $\dot{u}(0) = 0$

Put values in eq (a)

$\Rightarrow u(t) = \frac{1}{24} \times \cos(20.02t) + 0$

$$\Rightarrow u(t) = \frac{1}{24}\cos(20.02t)$$

Equivalent static force at any time 't' is

$\Rightarrow f_s(t) = Ku(t) = \frac{90625\cos(20.02t)}{24}$

$f_s(t) = 7552.08 \times \cos(20.02t)$

Now

Aptitude of dynamic displacement U_o for undamped free vibration

$$\Rightarrow U_o = \sqrt{\left[(u(0))^2\right] + \left(u(0)/w_n\right)}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0} = \frac{1}{24} \text{ ft}$$

Aptitude of equivalent static force f_s

$$kU_o = 90625 \times \frac{1}{24}$$

$$\boxed{kU_o = 3776.04}$$

QUESTION NO. 02.

ANSWER:-

GIVEN DATA:

use required data from Question "01"

REQUIRED DATA:

1. develop & solve the eq of motion for vibrations resulting at free end.
2. develop eqs showing variation in equivalent static forces with time
3. Draw Graph to show the variation of displacement with time & variation of equivalent static forces with time.

SOLUTION:-

§ ζ (Damping ratio) for RCC

Min = 0.80 % , Avg = 1.3%

So we consider 1.3%

E.O.M for damped free vibration

$ku + c\dot{u} + m\ddot{u} = 0$ ——— ①

from prob ①

$k = 90625$ lb/ft.

$m = 240.06$ lb. sec²/ft.

$c = \zeta \times 2m\omega_n$

$c = 2 \times 240.06 \times 20 \times 0.013$

$c = 124.83$ lb.sec/ft.

Putting values of k, m, & c in eq ①

① ⇒ $90625u + 124.83\dot{u} + 240.06\ddot{u} = 0$

(3)

Solution to the EOM for damped free vibration.

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\xi\omega_n\right]\cdot \sin(\omega_D t)\right]$$

$\omega_D = 20$ rad/sec

$$u(t) = e^{-0.013 \times 20t}\left[1/24 \times \cos(20t) + \frac{1}{20} \times \left[0 + 1/24 \times 0.013 \times 20 \times \sin(20t)\right]\right]$$

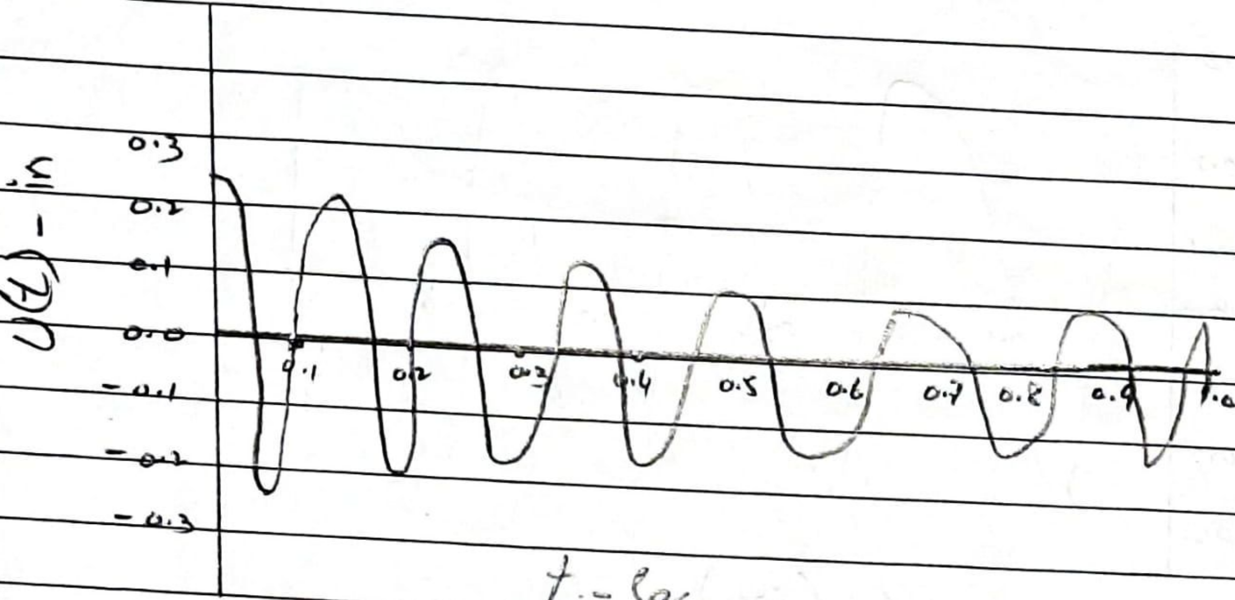
$$\boxed{u(t) = e^{-0.26t}\left[0.041 \times \cos(20t) + 0.0005416 \sin(20t)\right]}$$

$f_s(t) = k \cdot u(t) = 90625 \times u(t)$

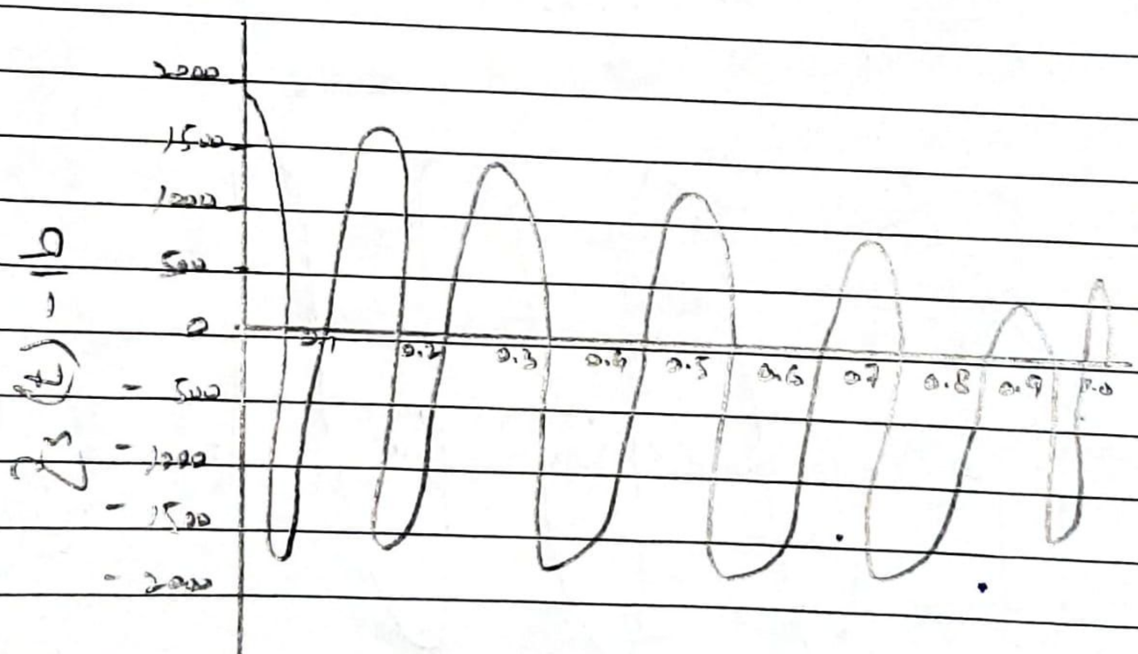
$$\boxed{f_s(t) = e^{-0.26t}\left[3715 \cos(20t) + 49.08 \sin(20t)\right]}$$

Date:

⇒ DAMPED FREE VIBRATION



⇒ DAMPED FREE VIBRATION



Question (3)

Given

⇒ Force = 60 kips

⇒ Displacement of tank = $\left[\frac{ID}{1000}\right]''$

$= \frac{7693}{1000} = 7.693''$

⇒ Time taken to complete 7 cycles = 3.57 sec

⇒ Amplitude of displacement = 2.286 cm = 0.9''

REQ

(a) Damping ratios

(b) Natural period of Undamped vibrations

(c) Stifness of structures

(d) Weight of tank

(e) Damping Co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5''

Solution

→ Displacement of tank $U_1 = 7.693''$
After 7 cycles
i.e After $j=7$, $U_{j+1} = U_8 = 0.9''$

ⓐ Damping ratios

$$\Rightarrow j = \frac{1}{2\pi\zeta} \ln\left[\frac{U_1}{U_{1+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.693}{0.9}\right]$$

$$\Rightarrow \zeta = \frac{1}{7}\left[\frac{1}{2\pi} \ln\left(\frac{7.693}{0.9}\right)\right] =$$

$\Rightarrow \zeta = 0.0459 = 4.59\% \Rightarrow$ $\boxed{\zeta = 4.59\%}$

ⓑ $T_D =$

→ As 7 cycles of vibrations completed in 3.57 sec

for one cycle, $T_D = \frac{3.57}{7} = 0.51$

$\boxed{T_D = 0.51 \text{ sec}}$ for one cycle

Now $W_D = W_n\sqrt{(1-\zeta^2)}$

$$\frac{2\pi}{W_D} = \frac{2\pi}{\left(W_n\sqrt{1-\zeta^2}\right)}$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{(1-\zeta)^2}} \Rightarrow T_n = T_D \times \sqrt{(1-\zeta)^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0.0459)^2}$$

$$\boxed{T_n = 0.51 sec}$$

(c) Stifness of structure

$$K = \frac{60 \times \cos 60^\circ}{\text{~~7.693~~}\ 2} = \text{~~3.894 K/in~~}$$

~~[illegible]~~ $K = 15\ K/in$

$$\boxed{K = 1800\ lb/ft}$$

(d) Weight of tank

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\frac{W}{g}}} = \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow W_n^2 = \frac{K \times g}{W}$$

$$\Rightarrow W = \frac{Kg}{W_n^2} \quad \text{Also } W_n = \frac{2\pi}{\tau_n}$$

$$W = \frac{Kg}{\left(\frac{4\pi^2}{T_n^2}\right)} = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = 1800 \times \frac{(0.51)^2}{4\pi^2}$$

$$32.2 \times 1800 \times \frac{0.2601}{12.56} = ~~3.7.9~~$$

~~W = 32.2~~ $32 \times 1800 \times \frac{0.0207}{12.56}$

$= 3.81k$

$$\boxed{W = 3818.64 \text{ lb}}$$

(e) $C = ?$

We know $\zeta = \frac{c}{2m\omega_n}$

$c = \zeta \times 2m\omega_n$

$C = \zeta\, 2m(2\pi T_n)$

$C = 0.0459 \times 2 \times \left[\frac{3818.6}{32.2}\right)\left(\frac{2\pi}{0.5}\right)$

$C = 0.0459 \times 237.1 \times 12.56$

$\boxed{C = 136.6 \text{ lb sec/ft}}$

(f) Number of cycles to reduce displacement amplitude

$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$

$J = \frac{1}{2\pi(0.0459)} \ln\left[\frac{7.693}{0.5}\right]$

$j = 0.288 \times (2.7) = 9$ cycles