Iqra National University, Peshawar
Department of Electrical Engineering

Final – Term Examination summer2020
Date:26/9/2020

Course Code:	MTH 203		**Course Title:**	Differential equation	
Prerequisite:	CALCULUS		**Instructor:**	HIMAYATULLAH	
Module:	3	**Program:** BEE	**Total Marks:**	80	**Time Allowed:**

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Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.		. Estimate general solution of $4y'' - 20y' + 25y = 0$.	Marks 15 PLO1 C2
Q2		Estimate the general solution of $y' = (x+2)y^2$	Marks 15
Q3		Calculat the initial value problem $y'' + 2y' + y = 0$ $y(0) = 4.$ $y'(0) = -6.$	Marks 10 PLO2 C4
Q4		Analyze the general solution of $x^2y'' + 3xy' + y = 0$.	Marks 10 PLO2 C4
Q5		**Examine** the method of undetermined coefficient method for $y'' + y' - 6y = 6x^3 - 3x^2 + 12x$.	Marks 10 PLO2 C4
Q6		**Examine** the method of variation of parameters for $y'' - 4y' + 4y = x^2e^{2x}$.	Marks 10 PLO2 C4

Q7		Identify an ODE $y'' + ay' + by = 0$ for the basis 1, e^{-3x}..	Marks 10 C4 PLO2

Q#1 Ans:

$4y'' - 20y' + 25y = 0$

$y'' + ay' + by = 0$

$4y'' - 20y' + 25y = 0$

÷ing by 4:

$\frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = 0.$

$y'' - 5y' + \frac{25}{4}y = 0$

$a = -5, b = 25/4.$

Auxiliary equ:

$\lambda^2 + a\lambda + b = 0$

$\lambda^2 - 5\lambda + \frac{25}{4} = 0.$

$\left(\lambda - \frac{5}{2}\right)^2 = 0.$

(2)

$$\left(\lambda + \frac{5}{2}\right)\left(\lambda - \frac{5}{2}\right) = 0$$

$$\lambda = \frac{5}{-2} \quad , \lambda = 5/2$$

Root are Real and

Distant :

$$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$$

$$\boxed{y = C_1 e^{-5/2\, x} + C_2 e^{5/2\, x}}$$

(2)

Q#2 Ans:

$y' = (x+2)y^2$

Sol:

As:

$y' = dy/dx$

$\frac{dy}{dx} = (x+2)y^2$

Taking integral w.r.t y:-

$\int dy/dx = \int (x+2)y^2 dy$

$\int \frac{d}{dx} y = \int xy^2 dy + \int 2y^2 dy$

$$y = \frac{xy^3}{3} + \frac{2y^3}{3} + c$$

(5)

Q#3 Ans:

$y'' + 2y' + y = 0$

$y(0) = 4 \quad , y'(0) = -6$

Sol:

Hamogonus equ:

$y'' + 2y' + y = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda+1) + 1(\lambda+1) = 0$

$\lambda(\lambda+1) + 1(\lambda+1) = 0.$

$(\lambda+1)(\lambda+1)$

$\lambda = -1, \lambda \neq -1.$

(5)

Root are Real and Equal:

$$y = C_1 e^{-x} + C_2 e^{-x}$$

$$y' = C_1 e^{-x} + C_2 e^{-x} - x e^{-x}$$

when = $y = 4$

$$y = C_1 e^{-x} + C_2 x e^{-x}$$

$$4 = C_2 e^{0} + C_2 e^{-1}$$

$$\boxed{4 = C_1} \quad \text{--- (i)}$$

when $x = 0$ $y = -6$

$$-6 = C_1 e^{0} + C_2 e^{-0} - 0 e^{-0}$$

$$\boxed{-6 = C_1 + C_2} \quad \text{--- (ii)}$$

Add 1 and 2:

$$4 = C_1$$
$$-6 = C_1 + C_2$$

$$\boxed{C_2 = -10}$$

(6)

Q# 4 Ans:

$x^2y'' + 3xy' + y = 0$

Sol:

$a = 3 \;, b = 1$

$m^2 + (a-1)m + b = 0$

$m^2 + (3-1)m + 1 = 0$

$m^2 (2)m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = -1, \; m = -1$

Root are Real and equal:

$y = (c_1 + c_2 \ln x) x^m$

(6)

(7)

$$y = (c_1 + c_2 \ln x) x^{-1}$$

(7)

Q# 5 : Ans:

$$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$$

Sol:

$$y'' + y' - 6y = 0$$

Auxiliary equ:

$$r^2 + ar + by = 0$$

$$r^2 + r + 6 = 0$$

$$r^2 + 3r - 2r - 6 = 0$$

$$r(r-3) - 2(r+3) = 0.$$

$$r = -3, \quad r = 2.$$

Root are Real and distand:

$$y = C_1 e^{-3x} + C_2 e^{2x}$$

choice of y_p:

$$y_p = K_3x^3 + K_2x^2 + K_1x + K_0$$

$$y_p' = 3K_3x^3 + 2K_2x + K_1$$

$$y_p' = 6K_3x - 2K_2$$

so:

$$6K_3x - 2K_2 + 3K_3x^2 + 2K_2x + K_1 - 6K_3x^3 - 6K_2x^3 - 6K_1x - 6K_2 = 6x^3 - 3x^2 + 12x$$

$$-6K_3 = 6$$

$$\boxed{K_3 = -1}$$

$$-6K_2 + 3K_0 = -3$$

$$-6K_2 + 3(-1) = -3$$

$$-6K_2 - 3 = -3$$

$$-6K_2 = -3 + 3$$

$$\boxed{K_2 = 6}$$

$$6K_3x + 2K_2 + K_1 = 12$$

$$6(-1) + 2(0) + K_1' = 12$$

$$-6 + K_1 = 12$$

$$\boxed{K_1 = 12}$$

(10)

$$-2K_1 + K_1 + K_0 = 0$$

$$-2(0) - 2 + K_0 = 0$$

$$\boxed{K_0 = 2}$$

(10)

Q#6 Ans:

$$y'' + 4y' + 4y = x^2 e^{2x}$$

Sol:-

$$y'' - 4y' + 4y = 0.$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda = 2, \lambda = 2.$$

Root are Real & equal:

$$y = (c_1 + c_2 x)e^{2x}$$

(12)

$y = c_1 e^{2x} + c_2 x e^{2x}$

$y_1 = e^{2x}$, $y_2 = x e^{2x}$

$y_1' = 2e^{2x}$, $y_1' = e^{2x} + 2x e^{2x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2 \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & e^{2x} + 2x e^{2x} \end{vmatrix}$$

$W = e^{4x} + 2x e^{4x} - 2x e^{4x}$

$\therefore \boxed{W = e^{4x}}$

So:

$$y_p = y_1 \int \frac{y_2 v(x)}{W} + y_2 \int \frac{y_1 v(x)}{W}$$

$$y_p = -e^{2x} \int \frac{x e^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx + x e^{2x} \int \frac{e^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx$$

(12)

(13)

$$y_p = e^{-2x} \int \frac{e^{4x} x^4}{e^{4x}} + x e^{2x} \int \frac{x^2 e^{4x}}{e^{4x}} dx$$

$$y_p = -e^{2x} \int x^2 dx + x e^{2x} \int x^2 dx$$

$$\boxed{y_p = -e^{2x} \cdot \frac{x^4}{4} + x e^{2x} \cdot \frac{x^3}{3}}$$

(13)

Q# 7 Ans:

$1, e^{-3x}$

Sol:

$1, e^{-3x}$

$y = c_1 e^{x} + c_2 e^{-3x}$

$\lambda = 0, \lambda = -3$

$\lambda - 0 = 0, \lambda + 3 = 0$

$\lambda(\lambda + 3) = 0$

$\lambda^2 + 3\lambda = 0$

$a = 3, b = 0$

$y'' - 3y' + y = 0.$

$$\boxed{y'' - 3y' = 0.}$$