Probability And Statistics

Sir Daud Khan

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I.D # 11453 BE(E)

Question No = 1

Student were asked how long it took them to walk to school on a particular morning. A cumulative frequency distribution was formed.

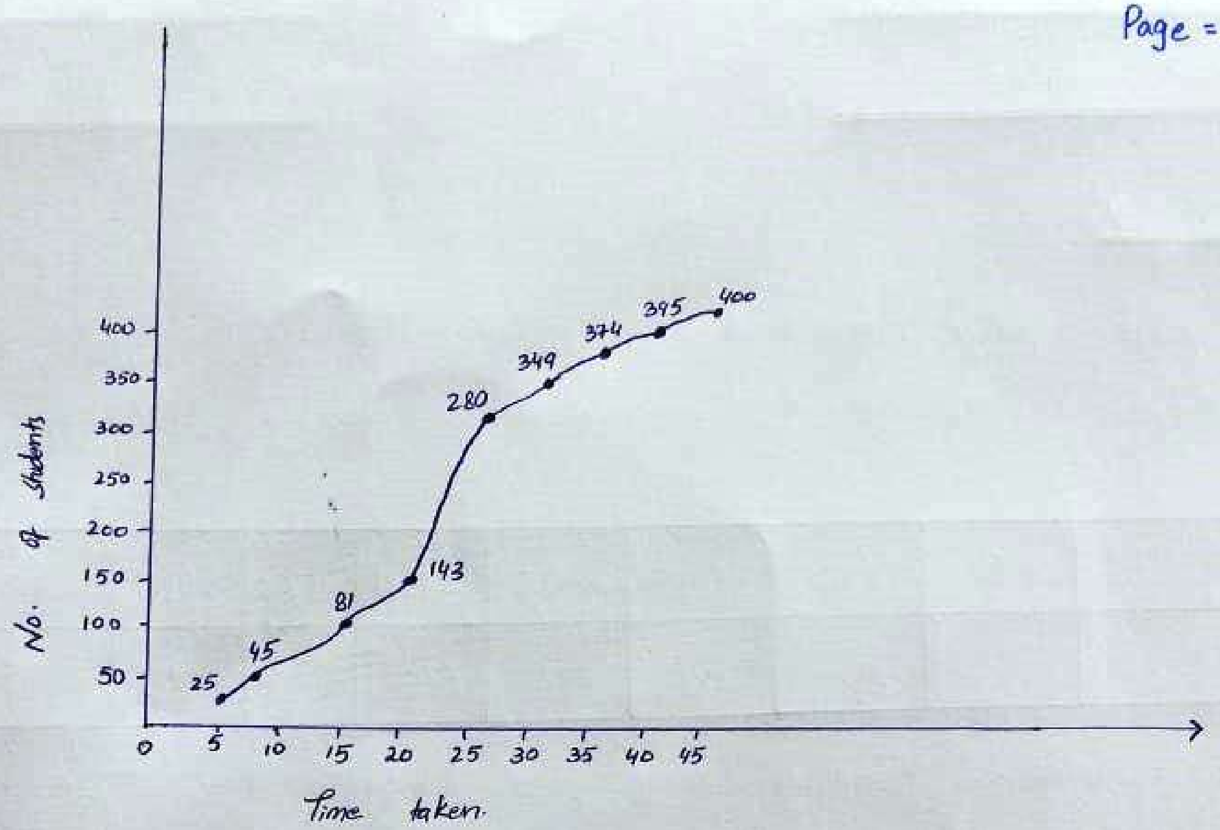
Time taken (in minutes)	<5	<10	<15	<20	<25	<30	<35	<40	<45
Frequency	25	45	81	143	280	349	374	395	400

a) Draw a cumulative frequency curve and estimate how many students took less than 18 minutes

b) Take equal class intervals of 0-, 5-, 10-, etc., construct frequency distribution and draw a histogram.

Answer Part (a)

Time Taken	C. Frequency	Frequency
<5	25	25
<10	45	20
<15	81	36
<20	143	62
<25	280	137
<30	349	69
<35	374	25
<40	395	21
<45	400	5
		400



$< 18 \quad = \text{Less tan } 15 + F_{20} \times 3/5$

$= 81 + 62 \times 3/5$

$= 81 + 37 \cdot 2$

$= 118 \cdot 2$

$= 118$ students

Answer

Q1 (b)

Take equal class intervals of 0-, 5-, 10-, etc construct frequency distribution and draw histogram.

Class	Class intervals	f	C.B	Class mark (Mid point x)
< 5	0 — 5	25	0.5 — 5.5	3
< 10	6 — 10	45	5.5 — 10.5	8
< 15	11 — 15	81	10.5 — 15.5	13
< 20	16 — 20	143	15.5 — 20.5	18
< 25	21 — 25	280	20.5 — 25.5	23
< 30	26 — 30	349	25.5 — 30.5	28
< 35	31 — 35	374	30.5 — 35.5	33
< 40	36 — 40	395	35.5 — 40.5	38
< 45	41 — 45	400	40.5 — 45.5	86

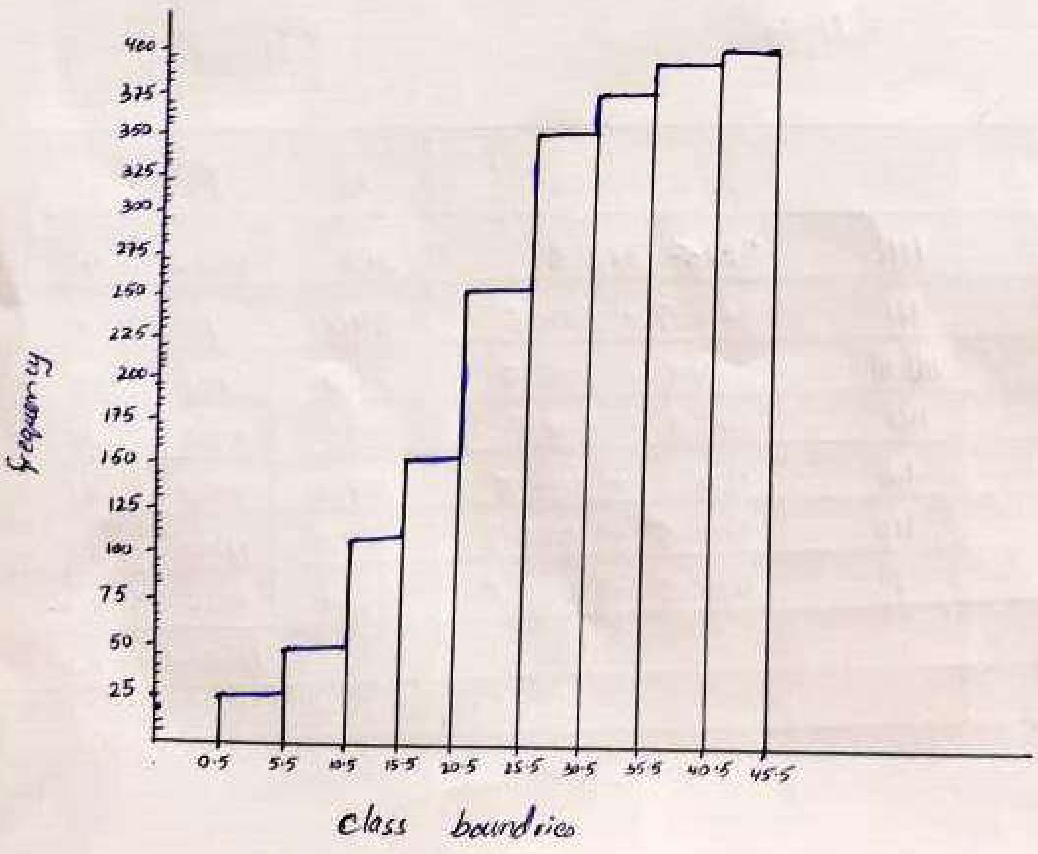
Scale:-

y-axis ⇒ 1 big square = 25 units

1 small square = 5 units

x-axis ⇒ 1 big square = 5 units

1 small square = 1 units



Question No: 2

Construct a grouped distribution table for the following data and calculate Mean, Mode and Quartiles.

423, 369, 387, 411, 393, 349, 371, 377, 389, 409, 392, 408, 431, 401, 363, 391, 405, 382, 400, 381, 399, 415, 428, 422, 396, 372, 410, 419, 386, 390

Solution:-

No. of observation = $N = 30$

largest value $= X_m = 431$

smallest value $= X_o = 363$

Now

Range $= X_m - X_o$

$= 431 - 363$

$R = 68$

Class intervales $= K = 1 + 3.33 \log(N)$

$= K = 1 + 3.33 \log(30)$

$= 1 + 3.33 (1.4771)$

$\boxed{K = 6}$

Class width $= C = \frac{R}{K} = \frac{68}{6}$

$= 11.33$

Class	frequency	tally	Class boundries	X	Fx	cf
363 - 373	4	||||	362.5 - 373.5	368	1472	4
374 - 384	3	|||	373.5 - 384.5	379	758	7
385 - 395	8	卌 |||	384.5 - 395.5	390	3120	15
396 - 406	5	卌	395.5 - 406.5	401	2005	20
407 - 417	5	卌	406.5 - 417.5	412	2060	25
418 - 428	4	||||	417.5 - 428.5	423	1692	29
429 - 439	1	|	428.5 - 438.5	434	434	30
Total	30					
					11,541	130

$$\text{Mean} = \bar{X} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{11,541}{30}$$

$$\boxed{\bar{X} = 384.7}$$

$$\text{Mode} = lb_{mo} + \left[\frac{D_1}{D_1 + D_2}\right] i$$

here,

$$lb = 385 - 0.5$$

$$\boxed{lb = 384.5}$$

$$i = 407 - 396$$

$$\boxed{i = 11}$$

$$\boxed{D_1 = 8 - 3 = 5}$$

$$\boxed{D_2 = 8 - 5 = 3}$$

$$\text{Mode} = 384.5 + \left[\frac{5}{5+3}\right] \times 11$$

$$= 384.5 + 6.875$$

$$\boxed{\text{Mode} = 391.375}$$

Quartile

→ Upper quartile $= Q_3 = l + \frac{\left(3/4\, N - f_c\right)}{f_Q} \times w$

$$3/4\, N = 3/4 \times 30$$

$$= 22$$

$$l = 406.5$$

$$f_c = 20$$

$$f_q = 5$$

$$w = 11$$

Putting the values

$$Q_3 = 406.5 + \left(\frac{22 - 20}{5}\right) \times 11$$

$$= 406.5 + 4.4$$

$$\boxed{Q_3 = 410.9}$$

→ Lower quartile $= Q_1 = l + \left(N/4 - f_c / f_q\right) \times W$

$N/4 = 30/4 = 7.5 = 7$

$l = 373.5$

$f_c = 4$

$f_q = 3$

$W = 11$

Putting values

$Q_1 = 373.5 + \left(7 - 4/3\right) \times 11$

$= 373.5 + 11$

$Q_1 = 384.5$

Inter quartile range $= IQR = Q_3 - Q_1$

$= 410.9 - 384.5$

$= 26.4$

Semi interquartile range $= SIQR = \frac{Q_3 - Q_1}{2}$

$= \frac{26.4}{2}$

$= 13.2$

Answer

Question No : 3

By multiplying each of the number 3,6,2,1,7,5 by 2 and then adding 5, we obtain 11, 17, 9, 7, 19, 15. what is the relation b/w standard deviation and the means of the two sets.

Solution :-

Given that

11, 17, 9, 7, 19, 15

Rearranging the data

7, 9, 11, 15, 17, 19

Mean :-

$$\text{Mean}, \bar{x} = \frac{\text{Sum of all}}{\text{Total no. of data}}$$

$$\bar{x} = \frac{7+9+11+15+17+19}{6}$$

$$\bar{x} = \frac{78}{6}$$

$$\Rightarrow \boxed{\bar{x} = 13}$$

Standard Deviation :-

$$S = \sqrt{\frac{(xi] + \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{(7-13)^2 + (9-13)^2 + (11-13)^2 + (15-13)^2 + (17-13)^2 + (19-13)^2}{6-1}}$$

$$S = \sqrt{22.4}$$

$\boxed{S = 4.73}$ → standard deviation.

So, it mean average of the data is 13, which deviates with a spread of 4.73 around the mean average.

Question No: 4

For the following grouped distribution table calculate the Variance and standard deviation.

Class	64-84	85-104	105-124	125-144	145-164	165-184	185-204
Frequency	15	18	27	10	6	5	13

Solution:-

Class	frequency	m	fm	$\bar{x}$	$m-\bar{x}$	$(m-\bar{x})^2$	$f(m-\bar{x})^2$
64 - 84	15	74	1,110	123.14	-49.14	2414.7	36,220
85 - 104	18	94.5	1,701	123.14	-28.64	820.2	1,47,636
105 - 124	27	114.5	3091.5	123.14	-8.64	74.6	2014.2
125 - 144	10	134.5	1,345	123.14	11.36	129	1,290
145 - 164	6	154.5	927	123.14	31.36	9883	5,898
165 - 184	5	174.5	872.5	123.14	51.36	2637	13,185
185 - 204	13	194.5	2,528.5	123.14	71.36	5092	66,195
	94	11,575.5					272438

$$\text{Standard deviation} = \sqrt{\frac{\sum f(m-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{272438}{94-1}}$$

$$= \sqrt{\frac{272438}{93}}$$

Standard deviation $= 54.12$

As,

$$\text{Standard deviation} = \sqrt{\text{Sample variance}}$$

So,

$$\text{Sample variance} = (\text{standard deviation})^2$$

$$= (54.12)^2$$

$$\boxed{\text{Standard variance} = 2,928}$$

Question No: 5

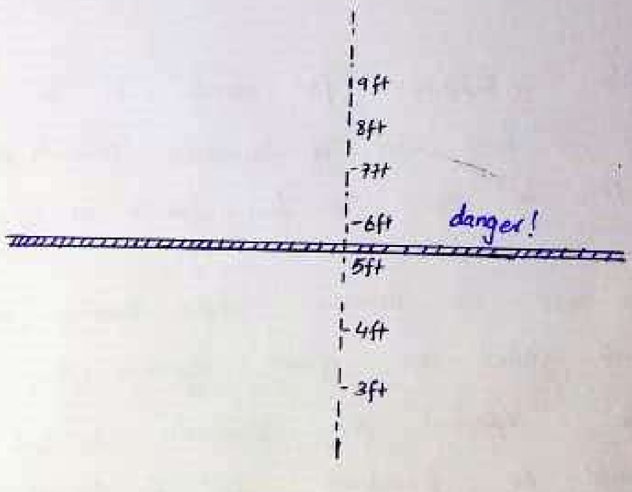
a) The depth of a river at four different points is 2, 7, 5, 6 feet respectively. The average depth is 5 feet. Therefore all the people with height 5 feet can across it.

Answer :-

No, not all the people of height 5 feet can cross the average 5 feet depth.

The first condition is that this is not compulsory for all 5 feet person that they can swim.

The most important fact is that a river of average 5 feet depth isn't deep uniformly. It can be 2 feet deep at one location and 1 feet at other. It can be 6 feet at one place or more feet at other places. So, even if a person is 6 feet tall, he can still drown in the river at average 5 feet depth. So, it is clear that not all the people of 5 feet can across the river (5 feet depth).



Part (b)

The average marks of one class of students are 30. Therefore every student is hopeless.

Answer :-

Average is basically a calculated central value of a set of numbers. So, if we say that a class of the students have average 30 marks, it isn't compulsory that all students are hopeless.

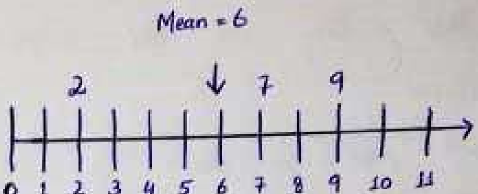
Average 30 marks doesn't mean that all students got the same 30 marks. Some may have got higher i-e 40, 50, 60 etc or

Some may have got less than 30 i-e 10, 20, 30 or even zero.

For example, the average of 2,7,9 is:

$$2+7+9 = 18 \quad , 18/3 = 6$$

So, average is 6 but if we look at the given 3 values, they include greater as well as lesser/smaller digits than 6 i-e 2,7 and 9. So, it is clear that all students aren't hopeless.



All this doesn't mean that averages are wrong. It only mean that we need more information to base our decision on.

Part (c)

The average income of a king and his household servants is £ 20,000 per month, therefore all the household servants must be fabulously paid.

Answer:-

As the income value of £20,000 per month is an average value of combined income of a king and of household servants, the ~~king~~ pay of different servants can be more or less, smaller or greater, minimum or maximum.

The average data is used to measure centre tendency in a data set. It works only when all values are equally important. Here in this case the pay amount can be different for different servants. So, we can't say that they should be fabulously paid. For example, you have a portfolio of stocks and it is highly unlikely that all stock will have same weight and therefore, the same impact on total performance of the portfolio. In finance, we have to work with unequal weights. In order to prevent any harm in decision making, we should be familiar with situation when it fails the assumption of fabulously paying them is unrealistic. Also, average is affected by extreme value in data set which in this case is £ 20,000. The mean may not coincide with any value of the salary of household servants.