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SECTION : A

SEMESTER : 4th BS (SE)

SUBJECT : CALCULUS AND ANALTYICAL GEOMETRY

INSTRUCTOR : ABRAR KHAN

EXAMINATION : MIDTERM PAPER

Q1)

a) Differentiate $\frac{3x^2 - 5x^2 + 5}{x^2 + 1}$ w.r.t x.

SOLUTION:-

By using formule

$\because \frac{d}{dx}\frac{f(x)}{f(g)} = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{(g(x))^2}$

$$\frac{3x^2 - 5x^2 + 5}{x^2 + 1}$$

$$= \frac{(x^2+1)(3x^2 - 5x^2 + 5)' - (3x^2 - 5x^2 + 5)(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x - 5x + 0) - (3x^2 - 5x^2 + 5)(2x)}{x^4 + 2x^2 + 1}$$

$$= \frac{\cancel{6x^3} - \cancel{10x^3} + 6x - \cancel{10x} - \cancel{6x^3} + \cancel{10x^3} - \cancel{10x}}{x^4 + 2x^2 + 1}$$

$$= \boxed{\frac{6x}{x^4 + 2x^2 + 1}} \text{ Ans}$$

Q.1)

b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ w.r.t x.

SOLUTION :-

By using formula :

$$\therefore \frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{(x^2+1)^2}{x^2-1} = \frac{x^4+2x^2+1}{x^2-1}$$

$$= \frac{(x^2-1)(x^4+2x^2+1)' - (x^4+2x^2+1)(x^2-1)'}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(4x^3+4x+0) - (x^4+2x^2+1)(2x-0)}{x^4-2x^2+1}$$

$$= \frac{4x^5+4x^3-4x^3-4x-(2x^5+4x^3+2x)}{x^4-2x^2+1}$$

$$= \frac{4x^5+4x^3-4x^3-4x-2x^5-4x^3-2x}{x^4-2x^2+1}$$

$$= \frac{4x^5-4x^3-2x^5+6x}{x^4-2x^2+1}$$

$$= \boxed{\frac{2x^5-4x^3+6x}{x^4-2x^2+1}}$$ Ans

Q2)

a) Find $\frac{dy}{dx}$ if $y = (1 + 2\sqrt{x})^3 \cdot x^{2/3}$ using chain rule.

$$y = (1+2\sqrt{x})^3 \cdot x^{2/3}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[(1+2\sqrt{x})^3 \cdot x^{2/3}\right]$$

$$\frac{dy}{dx} = (1+2\sqrt{x})^3 \cdot \frac{d}{dx}(x)^{2/3} + (x)^{2/3}$$

$$\frac{d}{dx}(1+2\sqrt{x})^3$$

$$= (1+2\sqrt{x})^3 \cdot 2 \cdot (x)^{2/3-1} + x^{2/3}$$

$$2\frac{d}{dx}(x)^{2/3}$$

$$= 2x^{-1/3}(1+2\sqrt{x})^3 + (x)^{2/3}(2)(2)(x^{-1/3})$$

$$= 2x^{-1/3}(1+2\sqrt{x})^3 + 2x^{2/3} \cdot 2x^{-1/3}$$

$$\boxed{\frac{dy}{dx} = 2x^{-1/3}\left[(1+2\sqrt{x})^3 + 2x^{2/3}\right]}$$

Ans

Q2)
b) Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1-x}{1+x}}$ using chain rule.

SOLUTION:-

Using chain rule formula

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \sqrt{\frac{1-x}{1+x}}$$

let $\frac{1-x}{1+x} = u$

$$y = \sqrt{u}$$

$$\frac{dy}{du} = \frac{1}{2} u^{1/2 - 1} = \frac{1}{2} u^{-1/2} \longrightarrow (1)$$

Now

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$\frac{du}{dx} = \frac{(1+x)\, d/dx\,(1-x) - (1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$= \frac{-1-x-1+x}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{-2}{(1+x)^2} \longrightarrow (2)$$

Now putting in formula:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2}u^{-1/2} \cdot \frac{-2}{(1+x)^2}$$

Now putting the value of u.

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \frac{-2}{(1+x)^2}$$

$$= -\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot (1+x)^{-2}$$

$$\boxed{\frac{dy}{dx} = -\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot (1+x)^{-2}}$$ Ans

Q3) a) Find the integration of $\int \frac{1}{\sqrt{x^3}} dx$.

SOLUTION:-

$$\int \frac{1}{\sqrt{x^3}} dx$$

let $x^3 = u$

$$\int \frac{1}{\sqrt{u} \cdot 3x^2} du$$

$$= \int \frac{1}{\sqrt{u} \cdot 3(u^{1/3})^2} du$$

$$= \int \frac{1}{3u^{2/3}\sqrt{u}} du$$

$$= \int \frac{1}{3u^{2/3}u^{1/2}} du$$

$$= \int \frac{1}{3u^{2/3+1/3}} du$$

$$= \int \frac{1}{3u^{7/6}} du$$

$$= \frac{1}{3}\int \frac{1}{u^{7/6}}\, du$$

$$= \frac{1}{3}\int u^{-7/6}\, du$$

$$= \frac{1}{3}\int \frac{u^{-7/6+1}}{-7/6+1} + c$$

$$= \frac{1}{3}\ \frac{u^{-1/6}}{-1/6} + \frac{1}{3}c$$

$$= -\frac{6}{3}\ \frac{1}{u^{1/6}} + \frac{1}{3}c$$

$$= -\frac{2}{u^{1/6}} + \frac{1}{3}c$$

$$= \frac{-2}{(x^3)^{1/6}} + \frac{1}{3}c$$

$$= \frac{-2}{x^{1/2}} + \frac{1}{3}c$$

$$\boxed{\frac{-2}{x^{1/2}} + \frac{1}{3}c}$$ Ans

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Q3
b) Find the integration of $\int \frac{1}{(6x+7)^6} dx$

SOLUTION:-

$$\int \frac{1}{(6x+7)^6} dx$$

let $6x+7 = u \longrightarrow (1)$

$$\frac{d}{dx}(6x+7) = \frac{du}{dx}$$

$$6 = \frac{du}{dx}$$

$$dx = \frac{1}{6} du \longrightarrow (2)$$

By putting (1) and (2) in question

$$= \int \frac{1}{u^6} \cdot \frac{1}{6} du$$

$$= \frac{1}{6} \int \frac{1}{u^6} du$$

$$= \frac{1}{6} \int u^{-6} du$$

$$= \frac{1}{6}\int \frac{u^{-6+1}}{-6+1} + C$$

$$= \frac{1}{6}\,\frac{u^{-5}}{-5} + \frac{1}{6}C$$

$$= -\frac{u^{-5}}{30} + \frac{C}{6}$$

$$= -\frac{(6x+7)^{-5}}{30} + \frac{C}{6}$$

$$= -\frac{1}{30(6x+7)^5} + \frac{C}{6}$$

$$= \boxed{-\frac{1}{30(6x+7)^5} + \frac{C}{6}}$$