Department of Electrical Engineering
Mid – Term Assignment Spring 2020
Date: 20/04/2020

Course Details

Course Title: Numerical Analysis **Module:** ______
Instructor: Sir Muhammad Waqas **Total Marks:** 30

Student Details

Name: Muhammad Maaz Akhunzada **Student ID:** 11448

Q1.	(a)	1. A 3x3 identity matrix has a total of 3 and ______ Eigen values. **a. <u>same</u>** b. different c. none d. zero 2. Eigen values of a symmetric matrix are all __________ **a. <u>real</u>** b. complex c. zero d. positive 3. All of the following are finite difference methods except for __________ a. Jacobi's method **b. <u>Newton's backward difference method</u>** c. Stirlling formula d. Forward difference method 4. The characteristics polynomial of a 3x3 identity matrix is ________, if x is the Eigen value of the 3x3 identity matrix. a. $(x-1)^3$ b. $(x+1)^3$ **c. <u>$x^3 - 1$</u>** d. $x^3 + 1$ 5. Two matrixes with the same characteristic polynomial does not need to be similar a. True **b. <u>false</u>** 6. Is the determinant of a diagonal matrix the product of the diagonal elements? **a. <u>true</u>** b. false 7. The Jacobi's method is a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal. **a. <u>No</u>** b. At least one c. At least two d. At least three 8. The power method can be used only to find the Eigen value of "A" that is largest in absolute value, we call this Eigen vale the dominant Eigen value of "A". **a. <u>true</u>** b. false 9. Central difference method is the finite difference method. **a. <u>True</u>** b. false 10. Iterative algorithms can be more rapid than direct methods. a. true **b. <u>false</u>** 11. $\Delta f_r = f_{r+1} - f_r$ is known as _________ difference operator. **a. <u>forward</u>** b. backward c. central d. none	Marks 11 CLO 1
Q2.	(a)	Use bisection method to solve the equation $x^2 - 7 = 0$, Perform four iterations and show all the necessary steps.	Marks 6 CLO 1
Q3.	(a)	Interpolate the value of 0.25 using Newton's forward difference formula. Show all the necessary steps. X: 0.2, 0.3, 0.4, 0.5, 0.6 F(x): 0.2304, 0.2788, 0.3222, 0.3617, 0.3979	Marks 6 CLO 1
	(b)	Use Newton Raphson method to find root of $f(x) = x^3 - 2x + 2$ *with* $x_0 = 0.2$. Perform four iterations.	Marks 6 CLO 1

X	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

Name : Muhammad Maaz Akhunzada

Q No :1 ID: 11448

Part (A)

Multiple Choice Questions:

1. A 3x3 identity matrix has a total of 3 and _______ Eigen values.
   a. same b. different
   c. none d. zero
2. Eigen values of a symmetric matrix are all __________
   a. real b. complex
   c. zero d. positive
3. All of the following are finite difference methods except for ____________
   a. Jacobi's method b. Newton's backward difference method
   c. Stirlling formula d. Forward difference method
4. The characteristics polynomial of a 3x3 identity matrix is ________, if x is the Eigen value of the 3x3 identity matrix.
   a. $(x-1)^3$ b. $(x+1)^3$
   c. $x^3 - 1$ d. $x^3 + 1$
5. Two matrixes with the same characteristic polynomial does not need to be similar
   a. true b. false
6. Is the determinant of a diagonal matrix the product of the diagonal elements?
   a. true b. false
7. The Jacobi's method is a method of solving a matrix equation on a matrix that has _____ zero along its main diagonal.
   a. No b. At least one
   c. At least two d. At least three
8. The power method can be used only to find the Eigen value of "A" that is largest in absolut value, we call this Eigen vale the dominant Eigen value of "A".
   a. true b. false
9. Central difference method is the finite difference method.
   a. true b. false
10. Iterative algorithms can be more rapid than direct methods.
    a. true b. false
11. $\Delta f_r = f_{r+1} - f_r$ is known as __________ difference operator.
    a. forward b. backward
    c. central d. none

QNo#2

(A) Use bisection method to solve the equation $x^2-7=0$. Perform four iterations and shows all the necessary steps:-

SOLUTION:-

$$F(x) = x^2-7=0$$

STEP NO 1:-

Assume Limits {Lower, Upper}

Limits = $[\underset{I}{1}, \underset{J}{2}]$

$$F(1) = (1)^2-(7) = -6$$

$$F(2) = (2)^2-7 = -3$$

$$F_1 \times F_2 = -6 \times -3$$

$$\boxed{= +18 > 0}$$

ITRATION:-

NEW LIMIT:-

$$[1, 3]$$

$F(3) = (3)^2 - 7 = 2$ | 11448 |

$F(1) \times F(3) = -6 \times 2$

$\boxed{= -12 < 0}$

STEP 2:-

If $F(l) \times F(U) =$ Ans < 0 then find mid point.

⇒ If $F(l) \times F(U) =$ Ans > 0 then change limit by finding.

Find $c = ?$ mid point

$$C = \frac{1+3}{2} = \frac{4}{2}$$

$\boxed{C = 2}$ Putting in the eq

$$\boxed{F(c) = (2)^2 - 7 = -3}$$

ITERATION 2

Now Limit [2, 3]

Mid Point = ?

$$C = \frac{2+3}{2} = \frac{5}{2}$$

$\boxed{= 2.5 \text{ put in eq}}$

$F(2.5) = (2.5)^2 - 7 = 6.25 - 7$

$= \boxed{-0.75}$

$F(2.5) \times F(3) = (-0.75) \times 2 =$

$\boxed{= -1.5 < 0}$

3y.

<u>STEP NO:- 3</u>

$[2.5\ 3]$

<u>ITERATION:-</u>

$C = \frac{2.5+3}{2} = \boxed{2.75}$ put in eq

$F(C) = (2.75)^2 - 7 \boxed{= -5.625}$

$F(C) \times F(U) = -5.625 \times 3 \boxed{= -4 < 0}$

$[2.75, 3]$

<u>Step 4:-</u>

<u>ITERATION:-</u>

$C = \frac{2.75+3}{2} = \boxed{2.875}$ put in eq

$F(C) = (2.875)^2 - 7 = 1.265$

$F(C) \times F(3) = 1.265 \times 2 = \boxed{2.5370}$

$F(C) \times F(2.75) = 1.265 \times -5625 = -7,115.6 < 0$

$[2.75, 2.875]$

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Pg# 05

Q No: 3

Part (A) Interplate the value of 0.25 using Newton's forward difference formula. Show all the necessary steps.

X	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

Sol:-

Step No 1:-

Difference Table:

X	y	Δ	Δ_2	Δ_3	Δ_4
0.2	0.2304	0.0484	-0.005	+0.0011	0.0005
0.3	0.2788	0.0434	-0.0039	0.0006	
0.4	0.3222	0.0395	-0.0033		
0.5	0.3617	0.0362			
0.6	0.3979				

Step 2

y_0	Δy_0	$\Delta_2 y_0$	$\Delta_3 y_0$	$\Delta_4 y_0$
0.2304	0.0484	-0.005	+0.0011	0.0005

STEP NO:- 3

$$X = a + nh$$

$$\Rightarrow X = 0.25$$

$$a \Rightarrow 0.2$$

$$h = 0.1 \quad [h = X_2 - X_1]$$

Putting the values:-

$$X = a + nh$$

$$0.25 = 0.2 + n(0.1)$$

$$0.25 - 0.20 = n(0.1)$$

$$0.05 = n(0.1)$$

STEP NO:- 4

Now find $n = ?$

$$n = \frac{0.05}{0.1} = 0.5$$

$$\boxed{n = 0.5}$$

$n = 0.5$ $y_0 = 0.2304$, $\Delta y_0 = 0.0484$, ~~$\Delta_2 y_0 = 0.00$~~

$\Delta_2 y_0 = -0.005$, $\Delta_3 y_0 = +0.0011$, $\Delta_4 y_0 = 0.0005$

STEP NO:- 5 Put the values In formula

$$f(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta_2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta_3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta_4 y_0$$

$$F(x) = 0.2304 + (0.5)(0.0484) + \frac{(0.5)(0.5-1)}{2\times1}(-0.005) + \frac{0.5(0.5-1)(0.5-2)}{3\times2\times1}(+0.0011) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{4\times3\times2\times1}(0.0005)$$

$$F(x) = 0.2304 + 0.0242 + \frac{(-0.25)}{2}\frac{(0.005)}{1} + \frac{(0.375)}{6}\left(\frac{0.0011}{1}\right) + \frac{(-0.9375)}{24}\frac{(-0.0005)}{1}$$

$$= 0.2304 + 0.0242 + 0.000625 + 0.00006875 + 0.0000195313$$

$$\boxed{= 0.2559320313}$$ Ans:-

QNO 3 Use Newton Raphson Method to find root
Part B of $f(x) = x^3 - 2x + 2$ with $x_0 = 0.2$ Perform four iterations.

Sol:- Using Newton Raphson formula:—

$$x_{i+1} = x_i - \frac{F(x_i)}{F'(x_i)}$$

Put $f(x_i)$ & $f'(x_i)$ values

$$x_{i+1} = \frac{x_i}{1} - \frac{(x_i^3 - 2x_i + 2)}{(3x_i^2 - 2)}$$

$$= \frac{x_i(3x_i^2-2)-(x_i^3-2x_i+2)}{3x_i^2-2}$$

$$= \frac{3x^3_i - 2x_i - x^3_i + 2x_i - 2}{3x^2_i - 2}$$

$$x_{i+1} = \frac{2x^3_i - 2}{3x^2_i - 2}$$

ITERATION 1:-

$x_0 = 0.2$

$$= \frac{2(0.2)^3-2}{3(0.2)^2-2}$$

$$= \frac{-1.984}{-1.88} = \boxed{1.0553}$$

ITERATION 2:-

$X_0 = 1.0553$

$$= \frac{2(x^3_i)-2}{3(x^2_i)-2}$$

$$= \frac{2(1.0553)^3-2}{3(1.0553)^2-2} = \frac{0.3504}{1.3409} = \boxed{0.2613}$$

ITERATION 3:-

$X_0 = 0.2613$

$$= \frac{2x^3_i - 2}{3x^2_i - 2} = \frac{2(0.2613)^3-2}{3(0.2613)^2-2}$$

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$$= \frac{+1.9643}{+1.7951}$$

$$\boxed{= \quad 1.0942}$$

ITERATION:-4

$$X_0 = 1.0942$$

$$= \frac{2x_i^3 - 2}{3x_i^2 - 2}$$

$$= \frac{2(1.0942)^3 - 2}{3(1.0942)^2 - 2}$$

$$= \frac{0.6201}{1.5918}$$

$$= \boxed{0.3895}$$ Ans.