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Subject : Differential equation.

Question no 1

Q Find the Fourier series .....

$$f(t) = 1+t, \ -\pi \leq t \leq \pi$$

Sol

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad \ldots (1)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}(2\pi + \pi^2)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n} \frac{d}{dt}(1+t) \right) \right)$$

$$a_n = \frac{1}{\pi}\left(\cancel{(1+t)\frac{\sin nt}{n}}^{\,0} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\sin nt\, dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi} \sin nt - \int\left(\int \sin nt \frac{d}{dt}(1+t)dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(-\frac{\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left[\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\cancel{\sin n\pi}^{0}}{n^2}\Big|_{-\pi}^{\pi}\right)\right]$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(\pi))(\cos n(\pi))\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

so equ becomes

$$\boxed{f(x) = \frac{1}{2\pi}(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin t}$$

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Question no 2

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Sol

step # 01

we have

$(A - \lambda I)x = 0$

A = Given matrix

Step # 02

we have: the characteristics equation is given by

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 03

$$\lambda^3 - \left|\text{sum of Diagonal Element}\right| \lambda^2 + \left|\text{sum of Diagonal minores}\right| \lambda - |A| = 0 \quad \text{---} (B)$$

sum of Diagonal elements $= 1 + 1 + 2 = 4$

Sum of diagonal minores $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

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by putting values in eqr (B);

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{---}(C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6 + 6$$

$$= 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Using Quadratic formula;

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \lambda = \frac{4-\sqrt{28}}{2}$$

We have eigenvalues;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ required solution

Question no 3

$5x + 4z + 2m = 3$

$x - y + 2z + m = 1$

$4x + y + 2z \quad = 1$

$x + y + z + m = 0$

Solu:

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & 9 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \begin{array}{l} \\ R_2 - 5R_1 \\ R_3 - 4R_1 \\ R_4 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \begin{array}{l} \\ 1/5 R_2 \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & -15 & -1 & -1 \\ 0 & 0 & -23/5 & 6/5 & -1/5 \end{array}\right] \begin{array}{l} \\ R_3 - 5R_2 \\ R_4 - 2R_2 \\ \\ \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & -23/5 & 6/5 & -1/15 \end{array}\right] (-1/15)R_3$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 113/75 & 8/75 \end{array}\right] R_4 + \frac{23}{5} R_3$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 1 & 8/113 \end{array}\right] (75/113)R_4$$

$\Rightarrow x - y + 2z + m = 1 \rightarrow (1)$

$y - 9/5 z - 3/5 m = -2/5 \rightarrow (2)$

$z + 1/15 m = 1/15 \rightarrow (3)$

$m = \frac{8}{113} \rightarrow (4)$

Rutting values of m in eqn (3)

$z + \frac{1}{15}(8/113) = 1/15$

$z + 8/1695 = 1/15$

$z = 1/15 - 8/1695$

$z = 113 - 8/1695$

$z = 105/1695$

Putting values in eq (2)

$y + 9/5(105/1695) - 3/5(8/113) = -2/5$

$$y + \frac{945}{8475} - \frac{24}{565} = \frac{-2}{5}$$

$$y + \frac{945 - 360}{8475} = \frac{-2}{5}$$

$$y + \frac{585}{8475} = -2/5$$

$$y = \frac{-2}{5} - \frac{585}{8475}$$

$$y = \frac{-3390 - 585}{8475}$$

$$y = \frac{-3975}{8475}$$

Putting values in eq (1)

$$x + \frac{3975}{8475} + 2\left(\frac{105}{1695}\right) + \frac{8}{113} = 1$$

$$x + \frac{3975}{8475} + \frac{210}{1695} + \frac{8}{113} = 1$$

$$x + \frac{3975 + 1050 + 600}{8475} = 1$$

$$x + \frac{5625}{8475} = 1$$

$$x = 1 - \frac{5625}{8475}$$

$$x = \frac{8475 - 5625}{8475}$$

$$x = \frac{2850}{8475}$$

$$x = \frac{570}{1695}$$

$$x = 114/339$$

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Question no 4

$U(x,t) = \sin(x+\gamma t)$

Sol:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$u(xt) = \sin(x+\gamma t)$ is solution of:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x}$$

it will statisfy the above equation

$$\frac{\partial u}{\partial t} = \cos(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+\gamma t)$$

again

$$\frac{\partial^2 u}{\partial t^2} = -2\sin(x+2t) \frac{\partial t}{dt}(x+2t)$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+\gamma t) \quad \ldots (a)}$$

Now $\frac{\partial u}{\partial x} = \cos(x+2t)$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \quad \ldots (b)}$$

comparing both A & B.

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$c = 2$

$\Rightarrow -4\sin(x+2t) = -c^2\sin(x+\gamma t)$

$\Rightarrow -4\sin(x+2t) + c^2\sin(x+\gamma t = 0$

This is possible if $c = \pm 2$

$-4\sin(x+\gamma t) + (\pm 2)^2 \sin(x+\gamma t) = 0$

$\boxed{0=0}$ This $u(x,t) = \sin(x+\gamma t)$