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(1)

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Sec A

Solution

1:2:1, 2:1:1, 2:0:2

40

P	E
A	E

B₁

50

P	P
A	E

B₂

P	P
A	A

B₃

Let x, y & z be the lost kg
Pak, Egyptian, American cotton respectively
then according to the given
conditions:

$$\frac{1}{4}x + \frac{2y}{5} + \frac{1}{4}z = 40$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50$$

$$\frac{2}{4}x + \frac{2}{4}z = 60$$

} \Rightarrow A

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} \rightarrow B$$

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$

$$AX = B$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

First $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times 0) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 0 - 1 \times 1)$$

$$= -2$$

Now

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} \text{ Exp by } R_1$$

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$|A_1| = -120$$

Similarity

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 200 & 1 & 1 \\ 120 & 1 & 1 \end{vmatrix} \text{ Exp by } R_1$$

$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix}$$

$$|A_2| = -40$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \text{ Exp by } R_1$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 160(0 - 1)$$

$$|A_3| = -120$$

Now according to Cramer's Rule

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

$$\text{Pakistan} = 60$$

$$\text{Egyptian} = 20$$

$$\text{American} = 60$$

