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PAPER CALCULAS AND ANALYTICAL Geometery

DEPARTMENT S.E (SEC A)

1): a) Let $y = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right)$$

$$\frac{dy}{dx} = \frac{(x^3+1)\frac{d}{dx}(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^6 - 6x^3 - 12x^3 - 6x^2 - 9x^6 + 6x^3 - 15x^2}{(x^3+1)^2}$$

$$\frac{dy}{dx} = \frac{3x^6 - 12x^3 - 15x^2}{(x^3+1)^2}$$ Ans.

1) b): Let $y = \frac{(x^3+1)^2}{x^3-1}$.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$$

$$\frac{dy}{dx} = \frac{(x^3-1)\frac{d}{dx}(x^3+1)^2 - (x^3+1)^2\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3-1)\cdot 2(x^3+1)(3x^2) - (x^3+1)^2(3x^2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)\left(6x^2(x^3-1) - 3x^2(x^3+1)\right)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(6x^5 - 6x^2 - 3x^5 - 3x^2)}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3+1)(3x^5 - 9x^2)}{(x^3-1)^2}$$ Ans.

2):

$$y = \int \frac{1}{\sqrt{x^5}}\,dx = \int \frac{1}{(x^5)^{1/2}}\,dx$$

$$y = \int (x^5)^{-1/2}\,dx = \int x^{-5/2}\,dx.$$

$$y = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C$$

$$y = \frac{x^{-3/2}}{-3/2} + C$$

$$y = -\frac{2}{3}\,x^{-3/2} + C.$$

$$y = -\frac{2}{3}\,(x^3)^{-1/2} + C$$

$$y = \frac{-2}{3(x^3)^{1/2}} + C$$

$$y = \frac{-2}{3\sqrt{x^3}} + C$$

Ans.

Q2/ (b): Let $Y = \int \frac{1}{(8x+7)^8} dx$.

$$Y = \int (8x+7)^{-8} dx.$$

$Y = 1/8$ Let $8x+7 = t$.

$$8dx = dt.$$

$$dx = \frac{1}{8} dt.$$

$$Y = \int (t)^{-8} \frac{1}{8} dt$$

$$Y = \frac{1}{8} \int t^{-8} dt = \frac{1}{8} \frac{t^{-7}}{-7} + C$$

$$Y = \frac{-1}{56 t^7} + C$$

$$Y = \frac{-1}{56 (8x+7)^7} + C.$$

Ans.

(d) $\int \frac{-x+9}{2x^2-8x+6}\,dx = \int \frac{-x+9}{2x^2-6x-2x+6}\,dx = \int \frac{-x+9}{2x(x-3)-2(x-3)}\,dx$

$$= \int \frac{-x+9}{(x-3)(2x-2)}\,dx$$

By partial fraction

$$\frac{-x+9}{(x-3)(2x-1)} = \frac{A}{x-3} + \frac{B}{2x-2} \quad \rightarrow (i)$$

$$-x+9 = A(2x-2) + B(x-3)$$

if $x = 3$

$$-3+9 = A(2(3)-2) + B(3-3)$$

$$6 = 4A + 0 \Rightarrow A = \frac{6}{4} \Rightarrow A = \frac{3}{2}$$

if $x = 1$

$$-1+9 = A(2(1)-2) + B(1-3)$$

$$8 = 0 + (-2B) \Rightarrow B = -\frac{8}{3}$$ put in eq (i)

$$\frac{-x+9}{(x-3)(2x-1)} = \frac{3}{2(x-3)} - \frac{8}{3(2x-2)}$$

$$\int \left[\frac{3}{2(x-3)} - \frac{8}{3(2x-2)}\right] dx = \int \frac{3}{2(x-3)}\,dx - \int \frac{8}{3(2x-2)}\,dx$$

$$= \frac{3}{2}\int \frac{1}{x-3}\,dx - \frac{8}{3}\int \frac{1}{2x-2}\,dx$$

$$= \frac{3}{2}\ln(x-3) - \frac{8}{3}\cdot\frac{1}{2}\int \frac{1}{x-1}\,dx$$

$$= \frac{3}{2}\ln(x-3) - \frac{8}{6}\ln(x-1)$$

Ans

$$\int \frac{4x^2+8x}{(x^3)(x^2+2x+3)}\,dx = \int \frac{4x^2+8x}{(x^3)(x^2+2x+3)}\,dx$$

$$\frac{4x^2+8x}{(x^3)(x^2+2x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+2x+3}$$

$$4x^2+8x = A(x^2)(x^2+2x+3) + B(x)(x^2+2x+3) + C(x^2+2x+3) + (Dx+E)(x^3)$$

$$4x^2+8x = A(x^4+2x^3+3x^2) + B(x^3+2x^2+3x) + C(x^2+2x+3) + (Dx+E)x^3$$

$$4x^2+8x = Ax^4+2Ax^3+3Ax^2+Bx^3+2Bx^2+3Bx+Cx^2+2Cx+3C+Dx^4+Ex^3$$

Comparing coefficient of like terms

$A + D = 0 \rightarrow (i)$

$2A + B + E = 0 \rightarrow (ii)$

$3A + 2B + C = 4 \rightarrow (iii)$

$3B + 2C = 8 \rightarrow (iv)$

$3C = 0 \Rightarrow \boxed{C = 0}$ put (iv) $\Rightarrow 3B = 8 \Rightarrow \boxed{B = 8/3}$

Put $B = 8/3$ & $C = 0$ in (iii)

$3A + 2(8/3) + 0 = 4 \Rightarrow 3A = -16/3 + 4 \Rightarrow 3A = \frac{-16+12}{3}$

$\Rightarrow 3A = -4/3 \Rightarrow \boxed{A = -4/9}$ put in (i)

$\boxed{D = 4/9}$ put $A = -4/9$ & $B = 8/3$ in (ii)

$2(-4/9) + (8/3) + E = 0 \Rightarrow \frac{-8}{9} + \frac{8}{3} + E = 0$

$\Rightarrow E = \frac{8}{9} - \frac{8}{3} \Rightarrow E = \frac{8-24}{9} \Rightarrow \boxed{E = -16/9}$

$-x + 3\ln|x-5| - \ln|x+3| + C$

$$\int \frac{4x^2+8x}{x^3(x^2+2x+3)} = \int \left[\frac{-\frac{4}{9}}{x} + \frac{\frac{8}{3}}{x^2} + \frac{0}{x^3} + \frac{\frac{4}{9}x + \left(-\frac{16}{9}\right)}{x^2+2x+3} \right]$$

$$= \int -\frac{4}{9x}\,dx + \frac{8}{3}\int \frac{1}{x^2}\,dx + \int \frac{\frac{4x-16}{9}}{x^2+2x+3}\,dx$$

$$= -\frac{4}{9}\int \frac{1}{x}\,dx + \frac{8}{3}\int \frac{1}{x^2}\,dx + \frac{1}{9}\int \frac{4x-16}{x^2+2x+3}\,dx$$

$$= -\frac{4}{9}\ln|x| + \left(\frac{-8}{3x}\right) + \frac{2}{9}\int \frac{9x-8}{x^2+2x+3}\,dx$$

$$= -\frac{4}{9}\ln|x| - \frac{8}{3x} + \frac{2}{9}\ln|x^2+2x+3|$$

4) Sol:- a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$.

$\Rightarrow X = \begin{bmatrix} 5-3 & 1+1 \\ -3-2 & 1-2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$ Ans.

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b): $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 2-4+1 & 6-8-0 \\ 1-2-0 & 5+0-2 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

Ans.

c): $X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$ Ans.

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5) Sol → $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

$A^2 + BC = ?$

$A^2 = A \cdot A$

$A^2 = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$

$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 9-3 & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$

$= \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$ Ans.