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Q① Estimate general Solution of

$$4y'' - 20y' + 25y = 0$$

Solution

$$ay'' + by' + cy = 0$$

and the Solution for This is

$$y = e^{\lambda} \quad \text{——} ①$$

$$y = c_1 e^{\lambda} + c_2 + e^{\lambda}$$

Now

$$4\frac{d^2}{dx^2}(y) - 20\frac{d}{dx}(y) + 25(-y)) = 0 \quad Ⓐ$$

Put eq ① in eq Ⓐ

$$= 4\frac{d^2}{dx^2}(e^{\lambda x}) - \frac{20d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0$$

$$= \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \quad \text{——} B$$

Now eq B and eq ① in eq Ⓐ

(2)

$$4\lambda^2 e^{\lambda x} - 20 e^{\lambda x} + 25 e^{\lambda x} = 0$$

$$e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$$

$$e^{\lambda x} \neq 0$$

$$\Rightarrow 4x^2 - 20\lambda + 25 = 0$$

$$(2\lambda - 5)^2 = 0$$

$$\lambda = 5/2 \quad \text{or} \quad \lambda = 5/2$$

$$y(x) = y_1(x) + y_2(x)$$

$$\Rightarrow y(x) = c_1 e^{5/2x} + c_2 x e^{5/2x}$$

Q2 (a) Calculate The initial Value Ploblem

$$y'' + 2y + y = 0$$

$$y(0) = 4. \quad y''(0) = -6$$

Solution

$$y'' + 2y' + y = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda + 1) + 1(\lambda + 1) = 0$$

$$\lambda = -1, \quad \lambda = 1$$

Root and real and Equation

$$X = -1$$

$$y(x) = y_1(x) + y_2(x) = c_1 e^{-x} + c_2 e^{-x} x$$

$$y' = c_1 e^{-x} + c_2 e^{-x}$$

$$y' = c_1 e^{-x} + c_2 e^{-x} - x e^{-x}$$

Solve for The unknow Constant using The initial conditions

$$\frac{dy(x}{dx}$$

$$\frac{dy(x)}{dx} = \frac{d}{dx}(c_1 e^{-x} + c_2 e^{-x} x)$$

$$= c_1 e^{-x} + c_2 e^{-x} - c_2 e^{-x} x$$

(3A)

Now

$y = 4, \; x = 0$

Subsitute $y = 4$ into $y(x) = e^{x} c_1 + e^{-x} x c_2$

$4 = c_1$ — (1)

$c_1 = 4$

Now

$x = 0 \quad , y = -6$

$y'(0) = -6$

$-6 \; c_1 e^{-x} + c_2 e^{-x} - c_2 e^{-x} x$

$= -6 = c_1 + c_2 \Rightarrow$ (2)

Add (1) and (2)

$4 = \cancel{c_1}$

$-6 = -\cancel{c_1} + c_2$

$2 = c_2$

Now =

$c_1 = 4$

$c_2 = 2$

Subsitute $c_1 = 4$ and $c = -2$ into

$y' = c_1 e^{-x} + c_2 e^{-x} x$

$y' = -2e^{-x}(x-2)$

Ans

(2) (C) (B) Analyze The general Solution of

$x^2y'' + 3xy' + y = 0$

Solution

$a = 3 \quad , \quad b = 1$

$m^2 + (a-1)m + b = 0$

$m^2 + (3-1)m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$(m+1)(m+1) = 0$

$m = 1 \quad , \quad m = 1$

roots are real and equal so

$y = (C_1 + C_2 \ln x) x^m$

Q3 Examine the method of underlermine coeficient method for $y''+y'-6y = 6x^3-3x^2+12x$

Solution

$$y''+y'-6y = 6x^3-3x^2+12x$$

$$y''+y'-6y=0$$

Auxelary Equ

$$\lambda^2+a\lambda+by=0$$

$$\lambda^2+\lambda-6=0$$

$$\lambda^2+3\lambda-2\lambda-6=0$$

$$\lambda(\lambda-3)-2(\lambda+3)=0$$

$$\lambda+3=0, \ (\lambda-2)=0$$

$$\lambda=-3 \quad , \quad \lambda=2$$

Part are Real and distrima

$$y = C_1e^{-3x}+C_2e^{2x}$$

choice for y_1

$$y_p = K_3x^2+K_2x^2+K_1x+K_0$$

$$y_1' = 3K_3x^2+2K_2x+K_1$$

$$y_p'' = 6K_3^2x-2K_2$$

Putin (1)

$6k_3x - 2k_2 + 3k_3x^2 + 2k_3x + k_1 - 6k_3x^3 - 6kx^3$
$- 6k_2x^3 - 6k_1x - 6kx - 6x^3 - 3x^3 + 12x$

Comparing

$-6k_3 = 6$, $-6k_2 + 3k_1 = -3$

$\boxed{k_3 = -1}$, $-6k_2 + 3(-1) = -3$

, $-6k_2 - 3 = -3$

$\Rightarrow -6k_2 = -3+3$

, $6k_2 = 0$

$\boxed{k_2 = 0}$

$6k_3x + 2k_2 + k_1 = 12x$

$6(-1) + 2(0) + k = 12$

$-6 + k_1 = 12$

$\boxed{k_1 = -2}$

$-2k_2 + k_1 + k_0 = 0$

$-2(0) - 2 + k_0 = 0$

$\boxed{k_0 = 2}$

$k_0, k_1, k_2, k_3 = (2, -2, 0, -1)$

Q(4) Examine the method of Variation of Parameters for $y'' - 4y' + 4y = x^2 e^{2x}$

Solution

$$y'' - 4y' + 4y = x^2 e^{2x}$$

for equation:

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda = 2 = \lambda = 2$$

Roots are and Equation

$$y = (c_1 + c_2 x)e^{2x}$$

$$y_c = c_1 e^{2x} + c_2 x e^{2x}$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}$$

$$y_1' = 2e^{2x}, \quad y_2' = e^{2x} + 2e x^{2x}$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2ex^{2x} \end{vmatrix}$$

$$W = e^{4x} + \cancel{2}xe^{4} - 2\cancel{xe^{4}}$$

$$W = e^{4x}$$

$$y_p = y_1 \int \frac{y_2\, g(x)}{W} + y_2 \int y_1 \frac{g(x)}{W}$$

$$y_p = -e^{2x} \int \frac{xe^{2x} \cdot x^2 e^{2x}\, dx}{e^{4x}} + xe^{2x} \int \frac{e^{2x} x^2 e^{2x}}{e^{4x}}\, dx$$

$$y_p = -e^{2x} \int x \frac{x^3 \cancel{e^{4x}}}{\cancel{e^{4x}}} + xe^{2x} \int \frac{x^2 \cancel{e^{4x}}}{\cancel{e^{4x}}}\, dx$$

$$y_p = -e^{2x} \int x^2\, dx + xe^{2x} \int x^2\, dx$$

$$y_p = -e^{3x} \frac{x^4}{4} + xe^{3x} \frac{x^3}{3}$$

So

$$y = y_h + y_p$$

$$y = c_1 e^{2x} + c_2 x e^{2x} - e^{2x} \frac{x^4}{4} + x e^{2x} + \frac{x^3}{3}$$

⑤

Q Identify an ODE $y''+ay'+by=0$ for the basis $1, e^{-3x}$

<u>Solution</u>

$$y_1 = e^{0x}, \quad y_2 = e^{3x}$$

$$y = C_1 e^{0x} + C_2 e^{3x}$$

So root are real and distinct

$$y = (C_1 e^{0x} + C_2 e^{\lambda_2 x})$$

So $\lambda_1 = 0$, $\lambda_2 = 3$

$$\lambda_1 = 0 \qquad \lambda_2 - 3 = 0$$

$$(\lambda)(\lambda - 3) = 0$$

$$\lambda^2 - 3\lambda = 0$$

So

$$\lambda^2 - a\lambda + b = 0$$

As $a = -3$, $b = 0$

So $y'' + ay' + by = 0$

$$y'' - 3y = 0$$

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