Q1 Find PQ where P is the point in three - dimensional space with coordinates (4,1,3) and the point Q with coordinates (1,2,4). Find the distance b/w P & Q. Further, Find the position vector of the point dividing PQ in the radio 1:3.

Sol

Coordinate of $P = (4,1,3)$

$$OP = 4i + 1j + 3k$$

$$OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \longrightarrow (1)$$

Now distance b/w P & Q $= |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \longrightarrow (2)$$

Let M be the point which divided PQ in ratio 1:3 Then by the ratio theorm position vector of

$$M = \vec{OM}$$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12j + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad — (3)$$

Hence eq 1, 2 & 3 are required Solution.

Q2) $\displaystyle\int \frac{4x^3+10x+4}{2x^2+x}\,dx$

Sol

$$
\begin{array}{r|l}
 & 2x-1 \\
2x^2+x & 4x^3+10x+4 \\
 & \pm 4x^3 \pm 2x^2 \\ \hline
 & -2x^2+10x+4 \\
 & \mp 2x^2 \mp x \\ \hline
 & 11x+4
\end{array}
$$

So $\displaystyle 2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = 2x-1+\int \frac{11x+4}{2x^2+x} \quad \text{— (1)}$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (2)$$

Now Find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (1)$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \longrightarrow (3)$$

Put $x=0$ in (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in (3)

$$11(-1/2)+4 = B(-1/2)$$

$$\frac{-11}{2} + 4 = \frac{-B}{2}$$

$$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$-3 = -B \Rightarrow \boxed{B=3}$$

Putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + 3/2 \ln|2x+1|$$

Putting these values in (2)

$$= x^2 - x + 4\ln|x| + 3/2 \ln|2x+1|$$

Now put These values in (1)

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2\ln|2x+1| + C$$

Q3/ (a) $\int_0^2 x^2 e^x \, dx$

Now find integration

$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$

$= x^2 e^x - 2 x e^x + 2 e^x$

Now put limits

$= \left(x^2 e^x - 2x e^x + 2 e^x \right)_0^2$

$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$

$= \left(4e^2 - 4e^2 + 2e^2 - 2 \right)$

$= 2e^2 - 2$ Ans.

Q3 b $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Sol

Let $u=\sqrt{x}$ so $du = \sqrt{x}/2x \; dx$

Limit at $x=1$ at $x=2$ $u=\sqrt{2}$

The original equation in variable is become.

$$= \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_1^{\sqrt{2}} \frac{\sin u}{u} \cdot 2u\,du$$

$$= 2\int_1^{\sqrt{2}} \frac{\cancel{u} \cdot \sin u\,du}{\cancel{u}}$$

$$= 2\int_1^{\sqrt{2}} \sin u\,du$$

$$= 2\left(-\cos u \Big|_1^{\sqrt{2}}\right)$$

$$= -2\left(\cos\sqrt{2} - \cos 0\right)$$

$$= 2\cos 1 - 2\cos\sqrt{2})$$ Ans.

Q4

The laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (A)$$

So $u(x,b,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x} = -\left[x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x} = 3x^2(x^2+y^2+z^2)^{-5/2} (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{2}(x^2+y^2+z^2)^{3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/2)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2}} - (2)$$

$$\frac{\partial u}{\partial z} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/3} - (x^2+y^2+z^2)^{-3/2} \quad - (3)$$

Putting ①, ② and ③ in Ⓐ

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+ 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-\cancel{y^2}-z^2+\cancel{3y^2}-\cancel{x^2}-\cancel{y^2}-z^2+3z^2-\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution of Laplace equation.