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Sumitted by :->

Question # 1 :->

The funcation $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t + 3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) state any point of discontinity,

(b) Find if they exist

(i) $\lim\limits_{t \to 3} g$

Solution :-> (a) To check possibility of the discontinity of the funcation is at $t = 0$ & 4

-> First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim g(1+h) = \lim (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 0^2 + 2(0)$$

$$= 1$$

→ For L.H.L

$$\lim_{h\to 0} g(1-h) = 2t+3$$

$$= \lim_{h\to 0} 2(1-h)+3.$$

$$= \lim_{h\to 0} 2-2h+3$$

Apply limit

$$= 2-2(0)+3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

→ Now at $t = 4$

$$g(4) = 2(4)+3$$

$$= 8+3$$

$$= 11$$

For R.H.L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$= \lim_{h\to 0} 2+2h+3$$

Apply limits

$$= 2+2(0)+3$$

$$= 5$$

(P-T-O) ⇒

For L.H.L

$\lim_{h \to 0} g(1+h) = 12$

$g(4) = R.H.L \neq L.H.L$

Point of discontinuity is at $t = 4$

(b) Find, if they exist

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2(3)$

$= 16$

→ L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

(P-T-O) ⟹

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exit since L.H.L is -ve)

Question # 2 :→ Find the Maclaurin's series for

$$y(x) = x^2 + \sin x$$

Solution:→ Since we know that the Maclaurin's series is

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)(x - x_0)^2}{2!} + \ldots$$

Put $x_0 = 0$

$$y(x) = y(0) + (x - 0)\,y'(0) + \frac{(x-0)^2 y''(0)}{2!} + \ldots$$

$$y(x) = y(0) + x y'(0) + \frac{x^2 y''(0)}{2!} + \ldots \to (1)$$

Now find

$y(0) = ?$

$$y(x) = x^2 + \sin x$$

$$y(0) = 0 + \sin 0$$

$$= 0 + 0$$

$$= 0$$

$$\boxed{y(0) = 0}$$

$$y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

P-T-O ⇒

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + \cos 0$$

$$= 0 + 1$$

$$\boxed{y'(0) = 1}$$

since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 - \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0$$

$$\boxed{y''(0) = 2}$$

Now

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

(P-T-O) ⇒

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(0) = -\cos 0$$

$$\boxed{y'''(0) = -1}$$

Put in eq 1

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots\cdots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots\cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots\cdots$$

So

$$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots\cdots$$

(A.1) Question #3 (a):→

$$1+xy=x^2+y^2 \quad , \quad \text{Find } y''=?$$

Solution:→ Given;

$$1+xy=x^2+y^2$$

Taking d/dx on both sides

$$1+\frac{d}{dx}x \cdot \frac{d}{dx}y = \frac{d}{dx}x^2+\frac{d}{dx}y^2$$

$$\Rightarrow \quad 1+(1)\left(\frac{dy}{dx}\right)=2x+2y\frac{dy}{dx}$$

$$1+\frac{dy}{dx}=2x+2y\frac{dy}{dx}$$

$$\frac{dy}{dx}-2y\frac{dy}{dx}=2x-1$$

$$\frac{dy}{dx}(1-2y)=2x-1$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2x-1}{1-2y}$$

$$\boxed{y'=\frac{2x-1}{1-2y}} \longrightarrow (1)$$

Diff Again

(P-T-O)⇒

(92)

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$$\frac{d}{dx}y' = \frac{d}{dx}\left(\frac{2x-1}{1-2y}\right)$$

$$y'' = \frac{(1-2y)\frac{d}{dx}(2x-1) - (2x-1)\frac{d}{dx}(1-2y)}{(1-2y)^2}$$

use Quoitent Rule.

$$= \frac{(1-2y)(2) - (2x-1)(-2y')}{(1-2y)^2}$$

$$= \frac{(2-4y) - (2x-1)(-2y')}{(1-2y)^2}$$

From eq (1) $y' = \frac{2x-1}{1-2y}$

Put in above

$$y'' = \frac{(2-4y) - (2x-1)\left(-2\left(\frac{2x-1}{1-2y}\right)\right)}{(1-2y)^2}$$

$$= \frac{2\cancel{(1-2y)}}{(1-2y)^{\cancel{2}}} - \frac{(2x-1)(2x-1)(-2)}{(1-2y)^3}.$$

$$\Rightarrow \boxed{\frac{2}{1-2y} - \frac{-2(2x-1)^2}{(1-2y)^3}}$$

Answer

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Question #3 (b):→

Find y' by using logarithmic differentiation.

$$y = x^3 (1+x)^9 e^{6x}$$

Solution:→ $\ln(y) = \ln\left(x^3 (1+x)^9 e^{6x}\right)$

$$= \ln\left(x^3 (1+x)^9\right) + \ln e^{(6x)}$$

$$= \ln x^3 + \ln(1+x)^9 + 6x$$

$$= 3\ln x + 9\ln(1+x) + 6x.$$

Now

$$\frac{d\ln(y)}{dx} = \frac{d}{dx}\left(3\ln x + 9\ln(1+x) + 6x\right).$$

$$= 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}.$$

$$= 3\cdot\frac{1}{x} + 9\cdot\frac{1}{1+x} + 6.$$

$$\boxed{\frac{d\ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6.}$$

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The End