Q1:-

=> ID # 7707

$Ku + Cu + Mu = P(t)$

In our case system is undamped $(c=0)$ undergoing free vibration $P(t)=0$

Hence General equation become EOM

$Ku + Mu = 0$

$K = 3EI/L^3$

$= 3 \times 29000 \times (150)$ in^4 $/ (10 \times 12)^3$

=> $K = 7.55$ k/in

$K = 90625$ lb/ft

=> $M = 7707$ lbsec^2 /

=> 239.34

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{90625}{239.34}}$$

$$\Rightarrow \omega_n = \boxed{19.445 \text{ rad/sec}}$$

$$\Rightarrow T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{20.030} =$$

$$T_n = \boxed{0.314 \text{ sec}}$$

=> Substituting The corresponding value in eq 1.

$$K u + M \ddot{u} = 0$$

$$90625 u + 239.34 \ddot{u} = 0$$

=> Where "K" is in lb/ft and "m" is in lb sec/ft².

=> General solution to The EQn for undamped free vibration is $u(t)$

$= u(0) \cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n} \sin(\omega_n t)$

$= 1/2 = \frac{1}{2} \times \frac{1}{12} = 1/24$ ft

and $\dot{u}(0) = 0$

=> $u(t) = (1/24)(\cos(19.445)) + 0$

=> $\left(\frac{1}{24}\right)(\cos(19.445))$

=> Equivalent static force at any time "t" is

$f_s(t) = k u(t) = \frac{90625 \times \cos(19.445)}{24}$

$f(s)(t) = 3776.04 \cos(19.445)$

3776

$f(s)(t) = 3560.66$

⇒ Amplitude of dynamic displacement

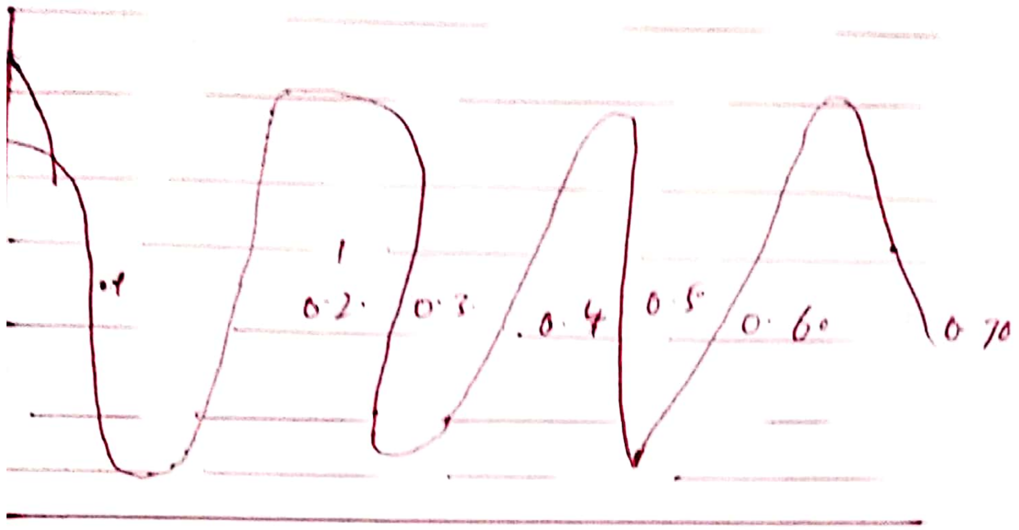
$u_0 = \sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2} = \sqrt{(1/24)^2 + 0}$

$= \sqrt{(1/24)^2}$

$= Ku_0 = 90625 \times \frac{1}{24}$

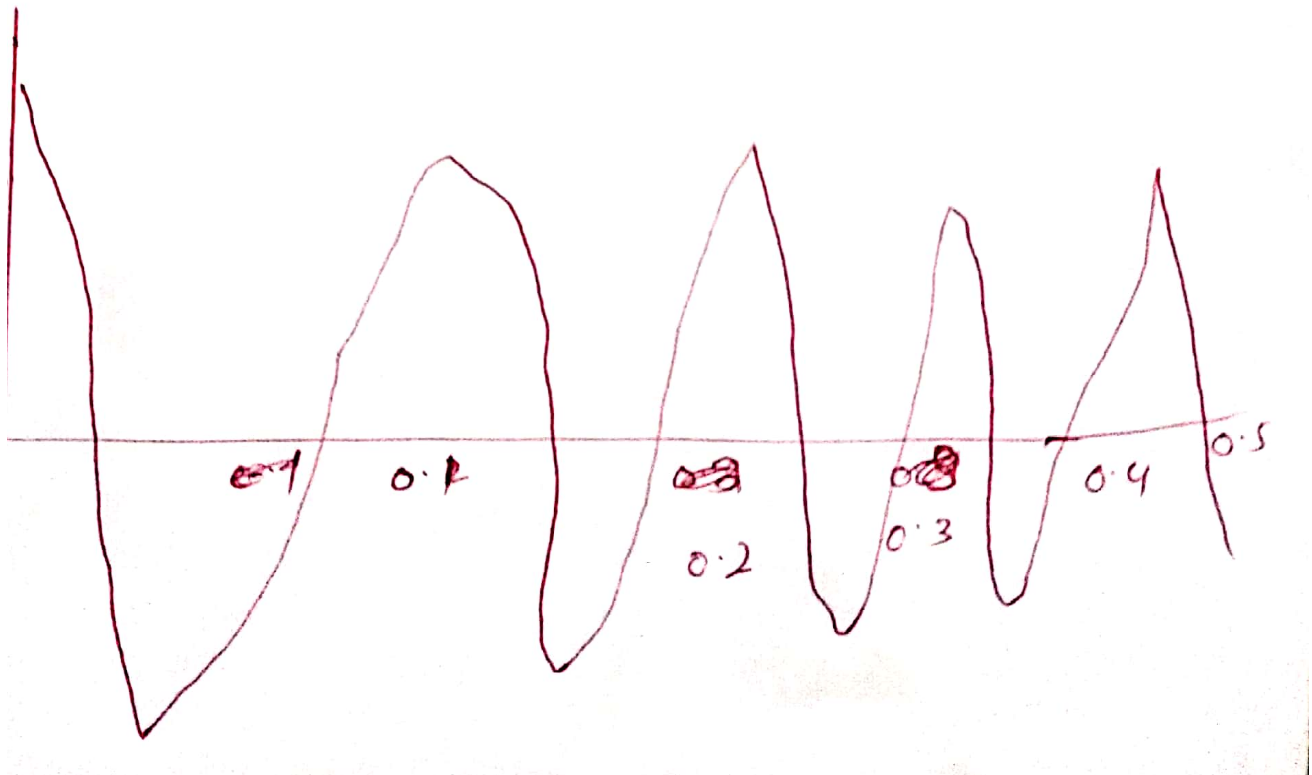
$= Ku_0 = 3776.041\ lb$

Undamped Force Vibration



t- sec

Undamped Free vibration.



Q3:- Given Data :-

Force = 60 kips

$u_1 = \frac{7707}{1000} = 7.707$ in

After $j = 7$ (cycles)

completed = 3.57 sec

u_{j+1} = 2.286 cm = 0.9 in

Ignore the vertical vibration.

Required :

(a) Dumping ratio

(b) Natural period of undamped vibration

(c) Natural of structure

(d) Weight of tank

(e) Damping co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5 "

Solution :-

u = Damping ratio = ?

As = $j = \frac{1}{2\pi u} \ln \left[{u_r}/{u_{j+1}} \right]$

By putting values

$$7 = \frac{1}{2(3.14)u} \ln \left[\frac{7.707}{0.9} \right]$$

$= u(7 \times 2 \times 3.14) = 2.147$

$= u(43.96) = 2.147$

$u = 2.147/43.96 = 0.0488$

(8)

$\Rightarrow$ $\zeta = 4.88\%$

$\Rightarrow$ (b) $T_D = ?$

As seven cycles are complet ted in "3.57" sec.

Thus time required to complete one cycle $= 7/3.57 = 1.96$ sec

$\Rightarrow$ $T_D = 1.96 \text{ sec}$

Now $\omega = \omega_n \sqrt{(1-\zeta^2)}$

$\Rightarrow$ $2\pi/\omega_n = 2\pi/\omega_n(\sqrt{1-\zeta^2})$

$\Rightarrow 1.96 \left(\sqrt{1-(0.0462)^2}\right)$

$T_n = 1.957$ " Natural period of undamped vibration "

(c) Stiffness of structure $K = ?$

$A_S = K \quad F\cos\theta / 2$

$\Rightarrow K = 60 \cdot \cos(60) / 2$

$F = 60$ kips

$\theta = 60°$

$\Rightarrow K = 15$ K/in

$\Rightarrow K = 18000$ lb/ft

(d) Weight of Tank $W = ?$

As $\omega_n = \sqrt{K/m} = \sqrt{K/w/g}$

$= \sqrt{K \cdot g / w}$

$\Rightarrow \omega_n^2 = K \cdot g / w \Rightarrow (w = K \cdot g / \omega_n^2)$

By putting value of $\omega_n = 2\pi / T_n$

$w = K \cdot g / (4\pi^2 / T_n^2) = K \cdot g \, (T_n^2 / 4\pi^2)$

$w = \frac{18000 \text{ lb}}{\text{ft}} \cdot \frac{32.2 \text{ ft}}{\text{sec}^2} \left(\frac{1.957}{4(3.14)^2} \right)$

$w = 56284.75$ lb

$= 56.284$ klb

(e) Damping co-efficient $c = ?$

It is know that $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow C = G(2m\omega_n) = G(2m(2\pi/k_n))$

· By putting value.

$C = 0.0462\left(2\left(\frac{56284}{32.2}\right)(2(3.14))\right)$

$C = 518.286$ lb/ sec/ft

(f) NO of cycles to reduce displacment attitude from "6.872 in to 0.5 in"

$j = ?$ $j = \frac{1}{2\pi h} \ln\left(\frac{u_1}{u_{j+1}}\right)$

$= \frac{1}{2(3.14)(0.0462)} \ln\left[\frac{7.707}{0.9}\right]$

$j = 7.39$

Q28

Solution: EOM for damped free vibration is

$$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow (1)$$

from question 1

$K = 90625 \text{ lb/ft}$

$m = 240.9 \text{ lb sec}^2/\text{ft}$

$\omega_n = 19.42 \text{ rad/sec}$

$c = \zeta \times 2m\omega_n$

$c = 0.025 \times 2(238.97)(20.04)$

$c = 0.025 \times 9577.9176$

$c = 239.447 \text{ lb/ft}$

Ku + cu + maü = 0

90625 + 239.4474u + 238.97 u = 0

Solution to The Eqn For damped Free vibration is

sin (wDt)

=> $w_D = \sqrt{k/m} = \sqrt{\frac{90625}{238.97}}$

=> $w_D =$ ' k/m

$\Rightarrow \omega_D = \sqrt{K/m} = \sqrt{\frac{90625}{239.34}}$

~~19.44~~ $\boxed{19.458 \text{ rad/sec}}$

$= u(t) = e^{-0.05 \times 19.45} \left[\frac{1}{24} \times \cos(19.45t) + \frac{1}{19.45} \times \left[0 + \frac{1}{24} \times 0.05 \times 19.45 \right] \times \sin(19.45) \right]$

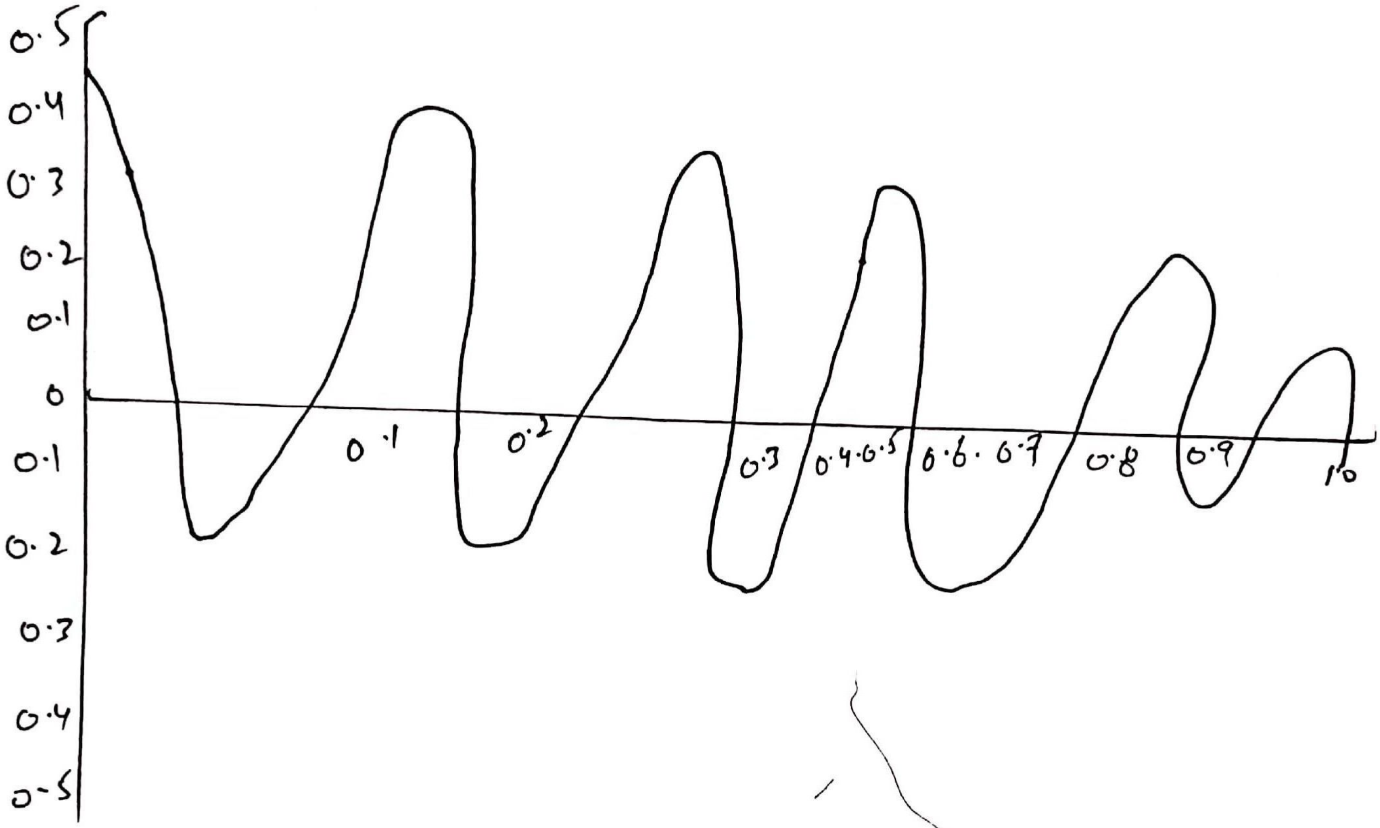
$\Rightarrow u(t) = e^{-0.972} \left[0.0416 \times \cos(19.45) + ~~0.041~~ \left[0.05 \right] \right.$

$= 0.0416 \times 0.05 \times 19.45 \sin 19.45$

$\Rightarrow$ ~~u(t)~~ $\Rightarrow f_{st} = k u(t) = 90625 \times u(t)$

$f_{st} = e^{-0.972} \left[3776 \cos(19.45) + 94.34 \sin(19.45) \right)$

Damped free vibration.



Damped free vibration.
t - sec