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Quiz 2

①

Question:-

A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian and American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kg of A, B, C costs 40, 50 ₹ or rupees respectively find the cost of a kg of cotton of each country.

Solution :- 1:2:1, 2:1:1, 2:0:2

40	
P	E
A	E

50	
P	P
A	E

P	P
A	A

Let my ₹ 2 be the cost 1 kg
 Pak, Egyptian American cotton
 respectively, then according to the given
 conditions

$$\begin{cases} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50 \\ \frac{2}{4}x + \frac{2}{4}z = 60 \end{cases} \rightarrow A.$$

$$\begin{cases} 1x + 2y + 1z = 160 \\ 2x + 1y + 1z = 200 \\ 1x + 1z = 120 \end{cases} \rightarrow B$$

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$AX = B$$

⇒

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

1st $|A|$

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \text{ expand by } R_1$$
$$= 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$
$$= 1(1 \times 1 - 1 \times 0) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 0 - 1 \times 1)$$
$$= -2$$

Now

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} \text{ expand by } R_1$$
$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$|A_1| = -120$$

Similarly



$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} \text{ expand by } R_1$$
$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 166 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$|A_2| = -40$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \text{ expand by } R_1$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 166 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 166(0 - 1)$$

$$|A_3| = -120$$

Now According to Cramer's Rule.

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistani = 60

Egyptian = 20

American = 60

