Q1 Solve the following objective type questions:

(i) The order of matrix $AB = m \times n$

(ii) Non zero rows in Echelon form is rank of matrix

(iii) $\begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

$|B| = 0 \Rightarrow |B| = 1 \times a - 4 \times 2 = 0$

$\Rightarrow a - 8 = 0 \Rightarrow a = 8$

(iv) $|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= (2i)(-i) - (i)(i)$

$= -2i^2 - i^2 \Rightarrow i^2 = -1$

(v) $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

This is Scaler Matrix

(vi) $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1 - 2x)$

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$y = e^{x-x^2} + c$$

(vii) $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\frac{dy}{dx}}$ is ?

Order = 1

degree = 6

(viii) $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is ?

Order = 2

Degree = Undefine

(ix) $2\frac{dy}{dx} + x^2y = 2x+3$

$$\int 2\,dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$2y = \frac{2x^2}{2} + 3x - y\,\frac{x^3}{3} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + c$$

Put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

(Homogenous equation)

So

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$$

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

R) $\begin{vmatrix} 1 & a & a^2 \\ 1-1 & b-a & b^2-a^2 \\ 1-1 & c-a & c^2-a^2 \end{vmatrix}$ $R_2 - R_1$, $R_3 - R_1$

$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$ expand by C_1

$$= 1\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} - 0 + 0$$

$$= \{(b-a)(c^2-a^2)\} - \{(b^2-a^2)(c-a)\}$$

$$= (b-a)(c-a)(c\cancel{-a}-b\cancel{+a})$$

$$= (b-a)(c-a)(c-b)$$

Q3 The rate of change in the form of differential equation is given by
$(x^2+3y^2)dx - 2xy\,dy = 0$.

Find the general solution at $x=2$, $y=6$

Sol:-

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2 \text{ and } y = 6$$

$$(x^2+3y^2)dx = 2xy\,dy$$

÷ing b/s by $2xy\,dx$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (i)$$

$$y = vx$$

$$dy = v\,dx + x\,dv$$

÷ing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \rightarrow \text{(A)}$$

Put A in eq(i)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{x}{vx} + \frac{3vx}{x}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

x'ing b/s by '2'

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

xing b/s by $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

xing b/s by $\frac{v}{x(1+v^2)}$

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

taking '$\int$' on b/s

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

taking 'e' on b/s

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$x^2+y^2 = x^3c \rightarrow$ (ii)

Put $x=2, y=6$ in eq (ii)

$(2)^2+(6)^2=(2)^2c$

$4+36=8c$

$40=8c$

$\frac{40}{8}=\frac{\cancel{8}c}{\cancel{8}}$

$c=5$

Put $c=5$ in eq (ii)

$x^2+y^2=5x^3$

$y^2=5x^3-x^2$

$y^2=x^2(5x-1)$

Taking Square root on b/s

$y^2=\pm\sqrt{x^2}\times\sqrt{(5x-1)}$,

$\sqrt{y^2}=\sqrt{x^2}\times\sqrt{(5x-1)}$

$y=\pm x\sqrt{5x-1}$

Ans

Q2 Find the Eigen value

(ii)
$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:
$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq $|A-\lambda I|=0$ Ⓐ

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Taking determinant

$$|A-\lambda I|=0$$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & 1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$2-\lambda\begin{vmatrix}3-\lambda & -1 & -1\\ -1 & 3-\lambda & -1\\ -1 & -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}$$

$$-1\begin{vmatrix}-1 & 3-\lambda & -1\\ -1 & -1 & -1\\ 0 & -1 & 2-\lambda\end{vmatrix} = 0 \rightarrow (i)$$

Again

$$\begin{vmatrix}3-\lambda & -1 & -1\\ -1 & 3-\lambda & -1\\ -1 & -1 & 2-\lambda\end{vmatrix}$$

Expand by R_1

$$\Rightarrow 3-\lambda\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - 1\begin{vmatrix}-1 & 3-\lambda\\ -1 & -1\end{vmatrix}$$

$$= (3-\lambda)\{(3-\lambda)(2-\lambda) - (-1)(-1)\} + 1\{(-1)(2-\lambda) - (-1)(-1)\} - 1\{(-1)(-1) - (-1)(3-\lambda)\}$$

$$=(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$
$$-1(1+3-\lambda)$$

$$=(3-\lambda)(\lambda^2-5x+5)+(-3+\lambda)-(4-\lambda)$$

$$=3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$=-\lambda^3+8\lambda^2-18\lambda+8 \to (a)$$

Now

$$+1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)$$

$$=-\lambda^2+5\lambda+\lambda-5-3$$

$$=-\lambda^2+6\lambda-8 \to (b)$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right)$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \rightarrow (c)$$

Putting (a), (b), (c) in eq (i)

$$\Rightarrow (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2$$
$$+6\lambda-8=0$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2$$
$$-8\lambda-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+18\lambda^2-\lambda^2-\lambda^2-36\lambda$$
$$-8\lambda+6\lambda+6\lambda+\cancel{16}-\cancel{8}-\cancel{8}=0$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

Now

$$
\begin{array}{c|cccc}
2 & 1 & -10 & 32 & -32 \\
 & & & -16 & 32 \\
\hline
 & 1 & -8 & 16 & 0
\end{array}
$$

$$(\lambda-2)(\lambda^3 - 8\lambda + 16\lambda) = 0$$

$$\Rightarrow \lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$$

$\lambda = 0$ | $\lambda - 2 = 0$, $\lambda = 2$ | $\lambda^2 - 8\lambda + 16 = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4) = 0$ | $(\lambda - 4 = 0$

$\lambda = 4$ | $\lambda = 4$

So

$\lambda_1 = 0$

$\lambda_2 = 2$

$\lambda_3 = 4$

$\lambda_4 = 4$

Ans

Q2 Express the determinant

(i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Sol:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + abc^2 + a^2b^3c - a^3b^2c$$

Taking (abc) common

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)$$

Ans