Name Zohaib Ahmad

ID 7797

Section A

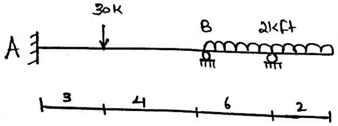
Subject Structural Analysis II

Exam Final Term

Scanned with CamScanner

7797

Q1 Analyze the beam shown in fig-1 by Stifness method Assume EI is constant.



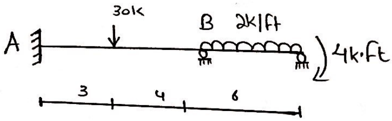
Solution

Step#1

Determining kinematic indetermanicy

K.I = 5°

So we have to reduce the extended portion



$$\Rightarrow \frac{2(2)}{1} = 4\text{k.ft}$$

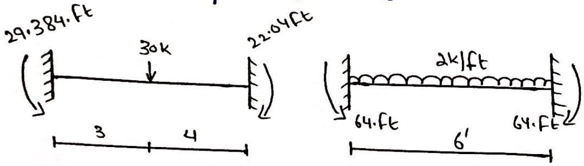
Now $U.I = 20$

Step 2 Determine unknown Displacement

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3

Compute [ADL] matrix



⇒ For pointed load (not at mid)

⇒ For left end

$$\frac{Pab^2}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

⇒ for UDL

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 64.\text{ft}$$

$ADL_1 = +22.04 - 6 = 16.04$ k.ft

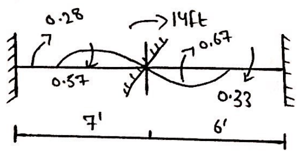
$ADL_2 = 6$ k.ft

Step # 4

Compute [S] matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = 1k$ $\quad D_2 = 0$



$\frac{4EI}{7} = 0.57$ | $\frac{2EI}{6} = 0.33$

$\frac{4EI}{6} = 0.67$ | $\frac{2EI}{7} = 0.28$

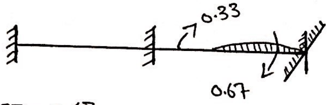
Scanned with CamScanner

$S_{11} = 0.57 + 0.67$

$= 1.24\,EA$

$S_{21} = 0.33\,EA$

(b) $D_1 = 0$, $D_2 = 1k$



$4EI = 0.67$

$2EI = 0.33$

$S_{12} = 0.33$

$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step 5

Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix} \times adj\,A$$

$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$

$= 0.8.308 - 0.1089$

$|S| = 0.7219$

$$Adj\,A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now :- $\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$

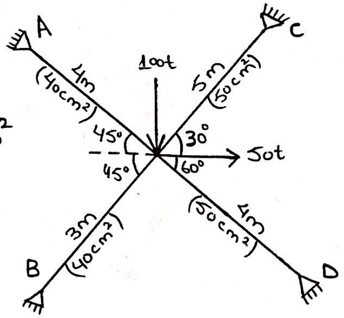
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} -10.08 \\ 2.81 \end{bmatrix}$$

Scanned with CamScanner

Q No(2)

$E = 2000t/cm^2$



Sol:-

For A:-

$\sin 45^\circ = P/h = P/4$

$\Rightarrow P = 2.828m$

$\cos 45^\circ = b/4$

$\Rightarrow b = 2.82m$

For B:-

$\sin 45^\circ = P/3$

$\Rightarrow P = 2.12m$

$\cos 45^\circ = b/h$

$\Rightarrow b = 2.12m$

For C:-

$\sin 30^\circ = P/h{=}5$

$\Rightarrow P \Rightarrow 2.5m$

$\cos 30^\circ = b/5$

$\Rightarrow b = 4.33m$

Scanned with CamScanner

Now :-

$EA_{(A)} = 2000 \times 40 = 80,000t$

$EA_{(B)} = 2000 \times 40 = 80,000t$

$EA_{(C)} = 2000 \times 50 = 100,000t$

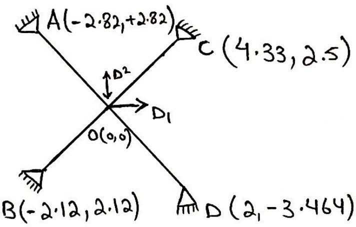
$EA_{(D)} = 2000 \times 50 = 100,000t$

Step:-1 K.I

$K.I = 2j - r$

$= 2(5) - 8 = 2^{\circ}$

Step :2 Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 3 $[AMD]_{4\times2}$ & $[S]_{2\times2}$

(i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(x_c - x_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0 - 433) = -173.2$$

Scanned with CamScanner

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0 - 200) = -125$$

Now $S_{11} = \sum^{3} \frac{EA}{L^3} (X_K - x_j)^2$

$$= \frac{80{,}000}{400^3} \times (282)^2 + \frac{80{,}000}{(300)^3} \times (212)^2 + 100{,}000 \times (-433)^2$$

$$+ \frac{100{,}000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 82.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum^{3} \frac{EA}{L^3} (X_K - X_j)(Y_K - Y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

(ii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2} (Y_K - Y_j)$$

$$AMD_{12} = \frac{80{,}000}{400^2} (-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2} (212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{500^2} (-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2} (346) = 216.25$$

Scanned with CamScanner

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3} (y_k - y_j)^2$

$$= \frac{80,000}{400^3}(-282)^2 + \frac{80,000}{300^3}(212)^2 + \frac{100,000}{500^3}(-250)^2$$

$$+ \frac{100,000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step:-4

$$[D] = [S]^{-1} \times [A \times D]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 44.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

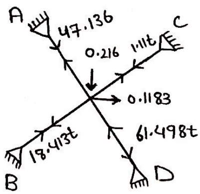
Step: [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

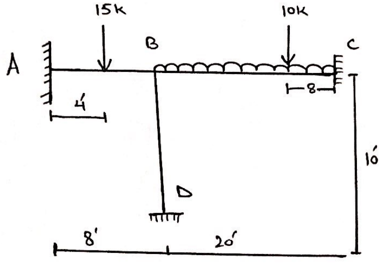
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Scanned with CamScanner

Q3 Analyze the rigid-joint frame shown in Fig 2 by .....?



Solution:-

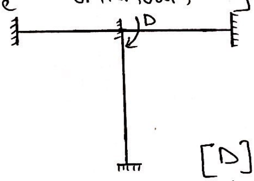
Step 1

Determine kinematic indetermenacy

$K.I = 1^{\circ}$

Step 2

Determine unknown joint Displacement

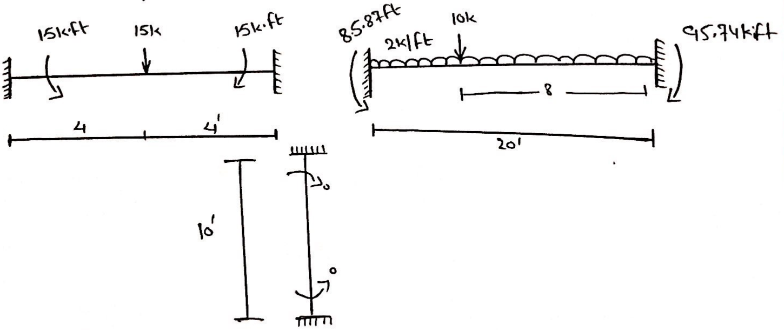


$[D] = [?]$

$[AD] = [0]$

Step:- 3

Compute [ADL] matrix



Point load at center

$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15\text{kip.ft}$$

$\Rightarrow$ uniformly Distributed load

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

Point load (Not at mid)

Suppose

For left end

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2\text{k.ft}$$

For Rigid End

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8\text{k.ft}$$

So total moment at left end

$$19.2 + 66.67 = 85.87\text{k.ft}$$

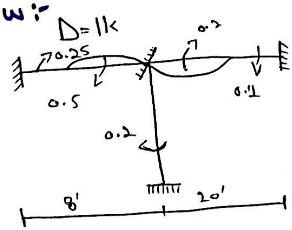
Similarly at right End

So $[AD] = -85.87 + 15 = -70.87\text{k.ft}$

Step 4 Determine [S] matrix

$$[S] = [S_{11}]$$

Now :-



$\Rightarrow \frac{4EI}{8} = 0.5$ $\qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2$ $\qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4$ $\qquad \frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\ EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step 5 :-

Compute (D) matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ {}^{1}/_{EI}$ Ans.

Scanned with CamScanner