Name: Sajid Ahmed Porgme: BECE)
ID: 12671 Subject: Numerical Analysis

1

Q 1 (a) Find the polynomial of degree 3 or less that interpolates the points (0, 2), (1, 1), (2, 0) and (3, -1)

Sol:

The lagrange form is as follows:

$$P(x) = 2\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 1\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$+0\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} - 1\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$= -\frac{1}{3}(x^3-6x^2+11x-6)+\frac{1}{2}(x^3-5x^2+6x)$$

$$-\frac{1}{6}(x^3-3x^2+2x)$$

$$\boxed{P(x) = -x+2}$$

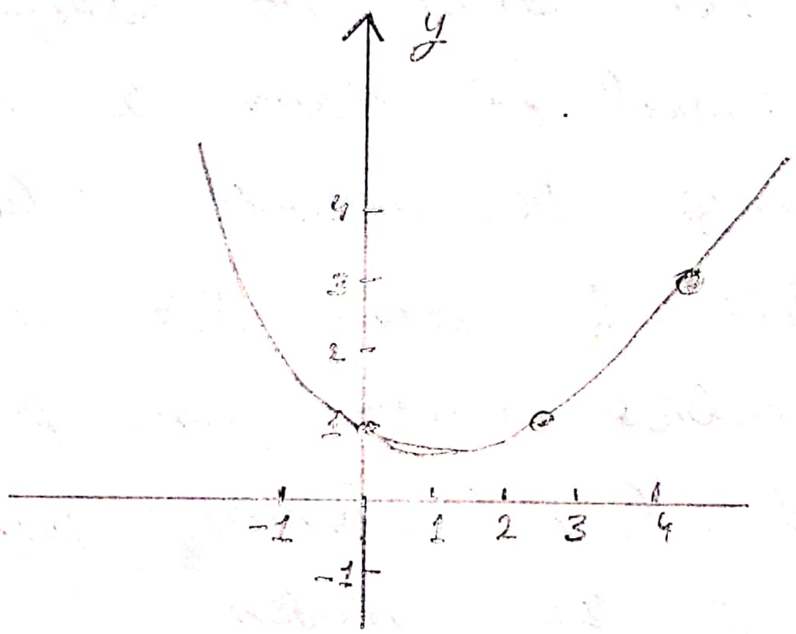
there exists exactly one interpolating polynomial of degree 3 or less, but it may or may

Name : Sajid Ahmad Programme : BE(E)
ID : 12671 2 Subject : Numerical Analysis

~~no~~ not be exactly degree 3.
the data points are collinear, so
the interpolating polynomial has degree
1. there ~~no~~ are no interpolating
polynomial of degree 2 or 3.
It may be already intuitively
obvious to you that no parabola
or cubic curve can pass
through four collinear points, but
here is the reason.

Name: Sajid Ahmad   Parogues   BE(E)
ID: 12671   3   Subject: Numerical Analysis

Q1 (b) Find an interpolating polynomial for the data points (0, 1), (2, 2) and (3, 4) for the figure given below.



Sol:→

Interpolation by parabola. The points as (0, 1), (2, 2) and (3, 4) are interpolated by the function

$$P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1.$$

Substituting into lagrange's formula

$$P_2(x) = 1\,\frac{(x-2)(x-3)}{(0-2)(0-3)} + 2\,\frac{(x-0)(x-3)}{(2-0)(2-3)}$$

$$+ 4\,\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

Scanned with CamScanner

Name : Sajid Ahmad   Paragrame : BE (E)

ID : 12671   4   Subject : Numerical Analysis

$$P_2(x) = \frac{1}{6}(x^2 - 5x + 6) + 2\left(-\frac{1}{2}\right)(x^2 - 3x) + 4\left(\frac{1}{3}\right)(x^2 - 2x)$$

$$= \frac{1}{2}x^2 - \frac{1}{2}x + 1$$

Check that $P_2(0) = 1$, $P_2(2) = 2$, and $P_2(3) = 4$,

Suppose that we presented with n points $(x_1, y_1) \ldots, (x_n, y_1)$. for each k b/w 1 and n, define the degree n-1 polynomial

$$L_k(x) = \frac{(x - x_1) \ldots (x - x_k) - 1)(x - x_k + 1) \ldots (x - x_n)}{(x_k - x_1) \ldots (x_k - x_k - 1)(x - x_k + 1) \ldots (x - x_n)}$$

the Interesting property of L_k is that $L_k(x_k) = 1$, while $L_k(x_j) = 0$, where x_j is any of the other data points. then ~~the~~ define the degree n-1 polynomial

$$P_{n-1}(x) = y_1 L_1(x) + \ldots + y_n L_n(x).$$

Name: Sajid Ahmad Programme: BE(E)

ID: 12671 5 Subject: Numerical Analysis

This is a straight forward generalisation of the polynomial and work same way Substituting x_K for x yields.

$P_{n-1}(x_K) = y_1 C(x_K) + \cdots y_n L_n(x_K) = 0$

$+ \cdots + 0 + y_K L_K(x_K) + 0 + \cdots + 0 = y_K$

So it works as designed.

Scanned with CamScanner

Name: Sajid Ahmad Program: BE(E)
ID: 12671 6 Subject: Numerical Analysis

Q2 (a) Use the two-point forward difference formula with $h=0.1$ to approximate the derivate of $f(x)=\frac{1}{x}$ at $x=2$.

Sol:→

Use formula

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

put the values.

$$f'(x) = \frac{f(2+0.1) - f(\frac{1}{2})}{0.1}$$

$$f'(x) = \frac{\frac{1}{2.1} - \frac{1}{2}}{0.1}$$

$$f'(x) = \frac{-0.02380}{0.1}$$

$$\boxed{f'(x) = 0.2380}$$

the difference b/w the approximation derivate.

$f'(x)$ $-x^{-2}$ at $x=2$ is the error

Name: Sayid Ahmad. Paragme: BE (E)
ID: 12671 (7) Subject: Numerical Analysis

put the values of $f'(x)$ and $x = 2$

$-0.2381 = -2^{-2}$

$-0.2381 = (-0.25)$

$-0.2381 - (-0.25)$

$\boxed{f'(x) = 0.0119}$

Now

$f''(x) = 2x^{-3}$ for c b/w 2 and 2.1

$f''(x) = 2(0.1)^{-3}$

$= 0.0125$

and

$(0.1)(2.1)^{-3}$

$(0.1)\ 0.10797 \Rightarrow 0.01079$

OR

$\boxed{= 0.0108}$

A second order formula According to Taylor's theorem.

Name: Sajid Ahmad Programe: BE(CE)
ID: 12671 8 Subject: Numerical Analy

$$f(x+h)=f(x)+hf'(x)+\frac{h^2}{2}f''(x)$$

$$f(2+0.1)=\frac{1}{2}+0.1)(0.0119)+\frac{(0.1)^2}{2}(0.0108)$$

$$f(2+0.1)=\frac{1}{2}+0.00119+\frac{0.01}{2}(0.0108)$$

$$f(2+0.1)=0.50119+0.0054$$

$$f(2+0.1)=0.50659$$

$$f(2.1)=0.50659$$

$$\boxed{f(x)=1.063839}$$

$$f(x-h)=f(x)-hf'(x)+\frac{h^2}{2}f''(x)$$

$$f(2-0.1)=\frac{1}{2}-0.1(0.0119)+\frac{(0.1)^2}{2}(0.0108)$$

$$f'(2.1)=\frac{1}{2}-0.00119+\frac{0.01}{2}(0.0108)$$

$$f'(2.1)=\frac{1}{2}-0.00119+0.0054$$

$$f'(2.1)=0.5-0.00119+0.00054$$

$$f'(2.1)=0.49881+0.00054$$

Name : Sajid Ahmad
ID : 12671

Programe: BE (E)
Subject: Numerical Analysis

$f'(2.1) = 0.49935$

$f'(x) = 0.49935\,(2.1)$

$\boxed{f'(x) = 1.048635}$

where

$x-h < c_2 < c_1 < x+h$

$f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

$f'(x) = \dfrac{f(2+0.1) - f(2-0.1)}{2(0.1)}$

$f'(x) = \dfrac{(2.1) - (1.9)}{0.2}$

$1.048635 = \dfrac{(2.1) - (1.9)}{0.2}$

$1.048635 = \dfrac{0.2}{0.2}$

$1.048635 = 1$

$1 - 1.048635$

$\boxed{f'(x) = -0.048635}$ Ans

Name : Sajid Ahmad
ID : 12671

Parogme : BE (E)
Module : 10th Semeste
Subject : Numerical An

10

Q 2 (b) Use Newton's divided differences to find the interpolations polynomial passing throug the points (0, 1), (2, 2) and (3, 4).

Sol:->

Apply the definitions of divided difference leads the following table.

0	1		
		$\frac{1}{2}$	
2	2		$\frac{1}{2}$
		2	
3	4		

This table is compute as follows After writing down the x and y coordinate in seperate colums. calculate the next colums. left to right as divided differences

$$\frac{2-1}{2-1} = \frac{1}{2}$$

$$\frac{2-\frac{1}{2}}{3-0} = \frac{1}{2}$$

$$\boxed{\frac{4-2}{3-2} = 2}$$

P.T.O

Name: Sajid Ahmad Programme: (BFCE)
ID: 12671 Subject: Numerial Analysis

(11)

After completing the divided difference triangle, the coefficients of polynomial $1, \frac{1}{2}, \frac{1}{2}$ can be read from the top edge of the table. the interpolating polynomial can be written as

$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}(x-0)(x-2)$$

Or insted form

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + (x-2)\cdot\frac{1}{2}\right)$$

the base points for the nested form are $x_1 = 0$ and $x_2 = 2$. Alternatively we could do more algebra and write the interpolating polynomials

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2) = \frac{1}{2}x^2 + 1.$$

Ans.

Q3 (a) Solve the least squares problem:

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

the normal equations $A^T Ax = A^T b$ are

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}$$

Solution of the normal equations are $\vec{x_1} = 3.8$ and $\vec{x_2} = 1.8$ the residual vector is

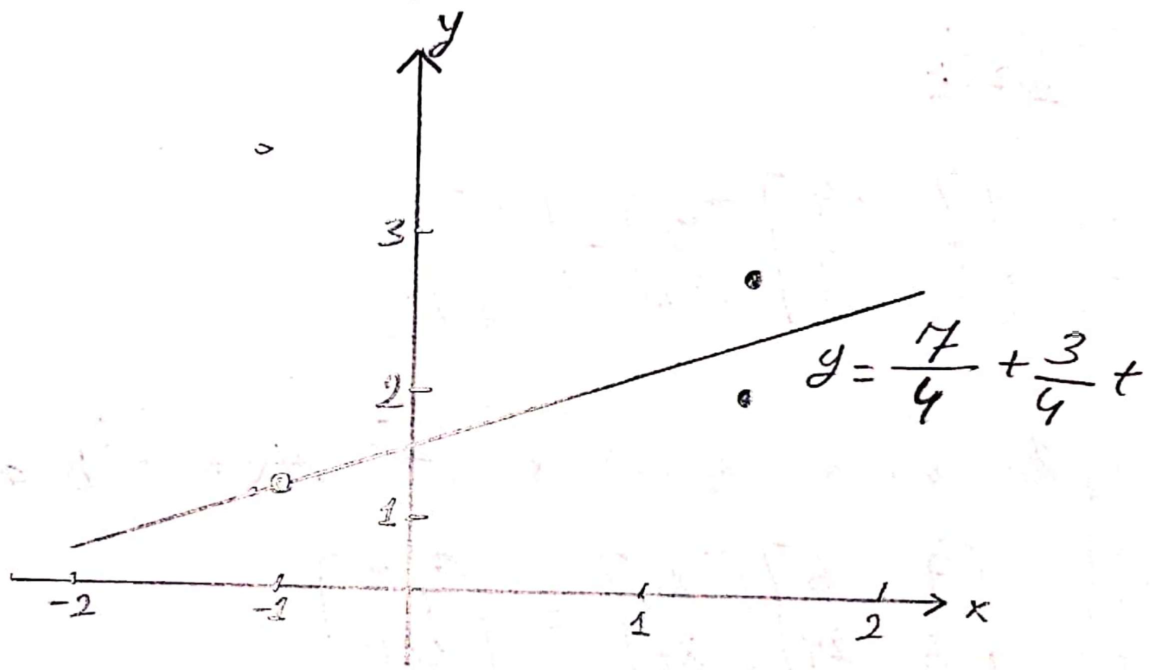
$$r = b - A\bar{x} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}$$

which has Euclidean norm $\|e\|_2$

$$= \sqrt{(0.4)^2 + 2^2 + (-2.2)^2} = \boxed{3} \text{ Ans}$$

Q3 (b) Find the line that best fits the three data points $(t, y) = (1, 2), (-1, 1)$ and $(1, 3)$ in the figure below.



Sol:→

The model is $y = c_1 + c_2 t$ and the goal is to find the best c_1 and c_2. Substitution of the data points into the model yields.

$$c_1 + c_2(1) = 2$$

$$c_1 + c_2(-1) = 1$$

$$c_1 + c_2(1) = 3$$

or in matrix form

P.T.O.

matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

We know that

This system has no solution (c_1, c_2) for two seperate reasons first, if there is a solution then the $y = c_1 + c_2 t$ would be a ~~linear~~ line containing the three data points. However it is easily seen that points are not collinear. this is the system of equation

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

and we ~~fand~~ found that the best solution in terms of least squres is $(c_1, c_2) = (\frac{7}{4}, \frac{3}{4})$ therefore the best lines is $y = \frac{7}{4} + \frac{3}{4}t$

we can evaluate the fit by using the statistics defined earlier. the residuals at the data point are.

t	y	line	error
1	2	2.5	-0.5
-1	1	1.0	0.0
1	3	2.5	0.5

and the RMSE is $\frac{1}{\sqrt{6}}$ as seen earlier.

the previous example suggets a three-step program for solving least squares data-fitting problems.

Fitting data by least squares.

Given a set m data points (t_1, y_1), ..., (t_m, y_m).

Step 1:→

Choose a model. Identify a parameterized model, such as $y = c_1 + c_2 t$, which will be used to fit the data

Name : Sajid Ahmad Program : BE (E)

ID : 12671 (16) Subject : Numerical Analysis

Step 2 :→

Force the model to fit the data: Substitute the data points into the model. Each data point creates an equation whose unknown are the parameters, such as c_1 and c_2 in the line model. this results in a system $Ax = b$, where the unknown x represents the unknown parameters.

Step 3 :→

Solve the normal equations. the least squares solution for the parameters will be found as the solution to the system of normal equations $A^T Ax = A^T b$