Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

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Q1.

(a) Consider the following analog signal (Marks 5, CLO 1)

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

i. Determine the minimum sampling rate required to avoid aliasing.

ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.

iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

(b) Consider a discrete time signal which is given by (Marks 5, CLO 1)

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$.

i. Draw the sampled signal.

ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .

iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.

Q2.

(a) Determine the response of the system to the following input signal with given impulse response (Marks 5, CLO 2)

$$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$

(b) Compute the convolution y(n) of the following signal (Marks 5, CLO 2)

$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$$

$$h(n) = \begin{cases} 2^n, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$$

Q3.

Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). (Marks 10, CLO 2)

i. $$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

ii. $$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & elsewhere \end{cases}$$

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Question # 1 Part ⓐ (1)

$3\cos 100\pi t + 4\sin 200\pi t$

ⓐ

i) Determind minimum Simpling rate to avoid alaising

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$$f_s \geq 2 f_{max}$$

$$f_1 = \frac{100\pi}{2\pi} = 50 HZ$$

$$f_2 = \frac{200\pi}{2\pi} = 100 HZ$$

Thus f_2 is maximum.

so $f_s = 2 \times f_{max}$

$$f_s = 2 \times 100$$

$$f_2 = 200 HZ$$

ii) Suppose that signal is sampled at rate $F_s = 100 HZ$ what is the discrit time signal obtaind after Sampling? Also explain the effect of this sampling newly generated discrit time signal.

$$F_s = 100 HZ$$

$$= \frac{100}{2} = 50 HZ$$

This is the maximum frequency which is representing by the sampled signal.

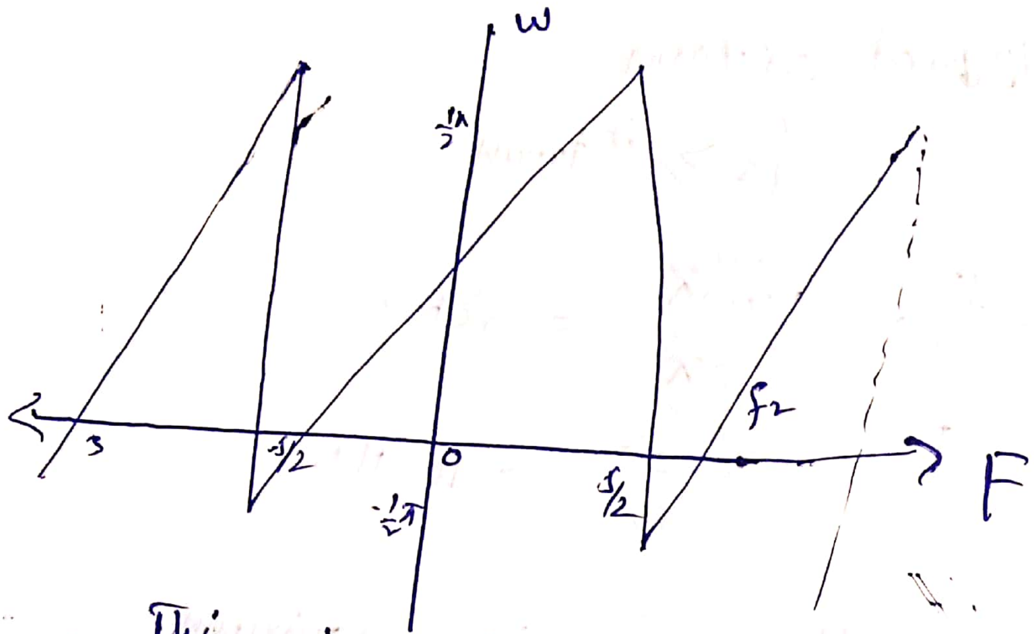
P-to

(2)

As

$$x_o(n) = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$3\cos 2\pi(5/10)n + 4\sin 2\pi n$$



This is the result of descrit ~~to~~ time signal.

iii) What is the anlog signal $y_a(t)$ we can reconstruct from the sample if we used Ideal Interpolation.

Sol:

folding freq of sampled signal

$$f_{fold} = F_s/2 = \frac{\cancel{100}^{50}}{\cancel{2}}$$

$$f_{fold} = 50\,Hz$$

and the frequency of orignal signal

Hence for Ideal Interpolation we can construct the orignal signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

we use a sampling frequency at Nayquist ~~Nyquist frequency~~ rate so the orignal signal is constructed.

Q 1: Part (b)

Consider discrit time signal which is given

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

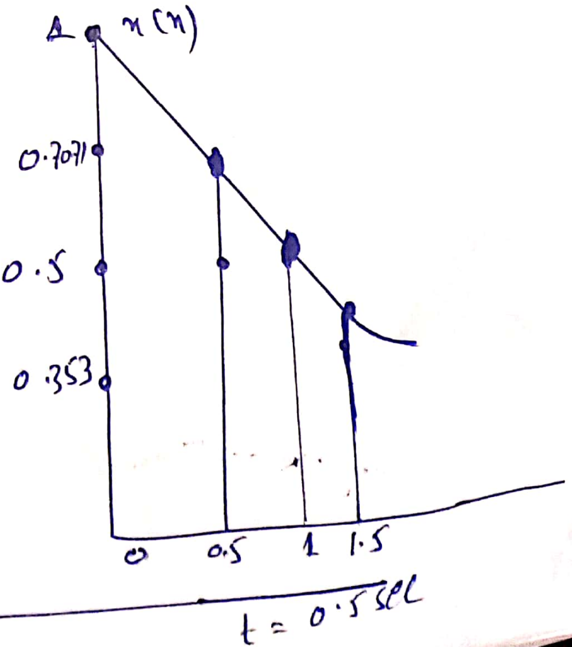
The signal is sampled at the rate $f_s = 2\,Hz$

$f_s = 2\,Hz$

$f_s = \frac{1}{T} = T = \frac{1}{f_s} = \frac{1}{2} = 0.5\,sec$

i) Draw the Sampled signal

$x(n)$	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



Q 2: (5) Q1 → Part (b)

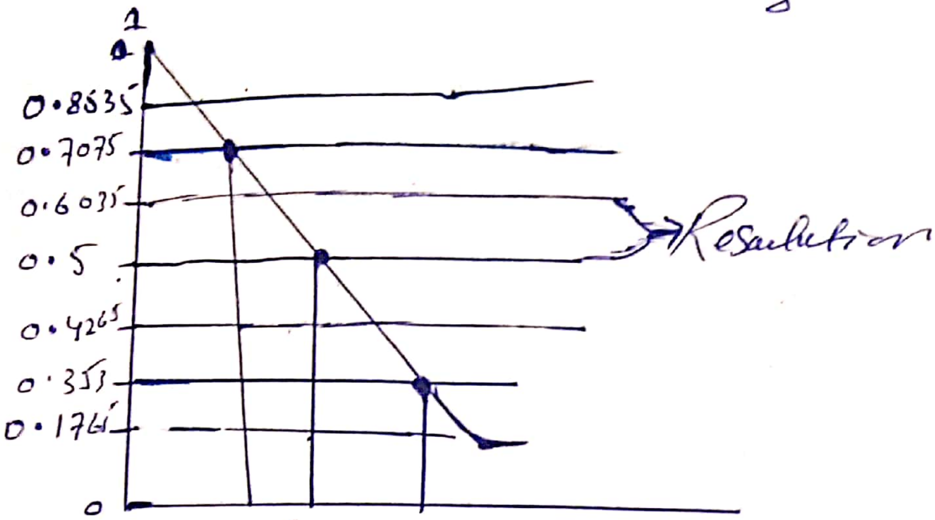
ii)

$L = 2^n$

n = number of bit = 3

$L = 2^3 = 8$ bits

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L} = \frac{1-0}{8} = 0.125$$



Quantization level.

1, Part (b)

iii)

	Discrit time signal	Trunction	Rounding	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.7076	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

Q2:

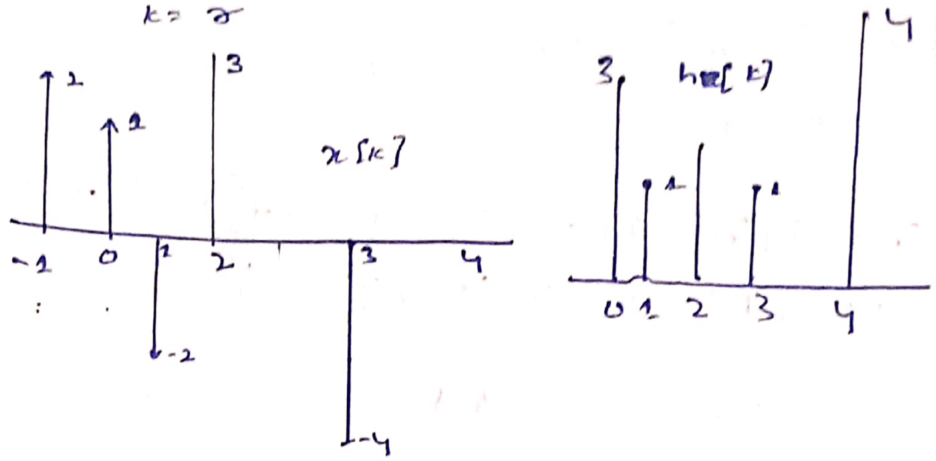
(b)

(a) $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$

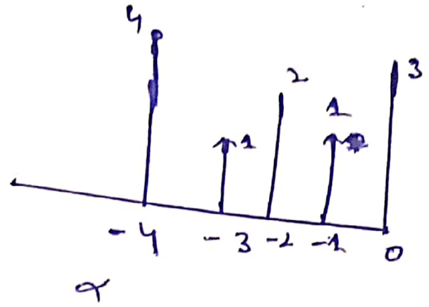
$h[n] = \{3, 1, 2, 1, 4\}$

Sol:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$



$h[-k]$ fold signal

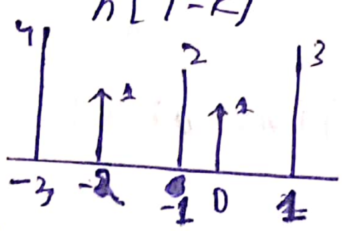


$$y[0] = \sum_{k=-1}^{\infty} x(-3)\, h(-1) + 1(0)\, h(0)$$

$2 \times 1 + (1)(3)$

$= 5$

for $n = 1$



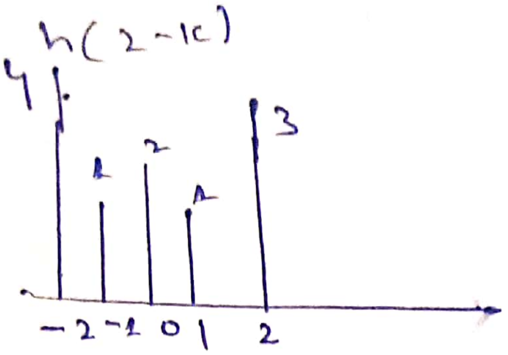
$$y[1] = \sum_{k=-1}^{1} x(n)\, h[1-k]$$

$x(-1)\, h(-1) + x(0)\, h(0) + x(1)\, h(1)$

$(2)(2) + 1(1) + (3)(-2) = 4 + 1 - 6 = -1$

$n=2$

7)



$$y[2] = \sum_{k=-1}^{2} x(n) . + h(2-k)$$

$= x(-1)\, h(-1) + x(0) + h(0) + x(1) + x(2)\, h(2)$

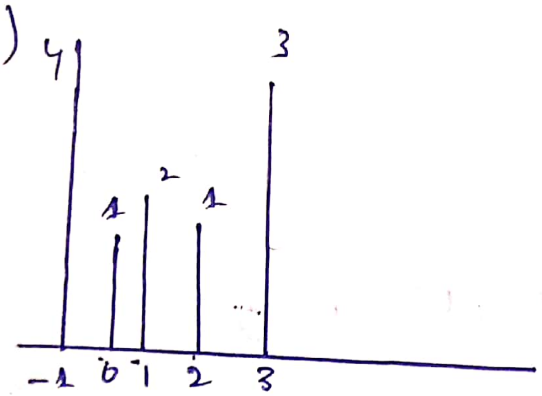
$2(1) + 1(2) + (-2)(1) + (3)(3)$

$= 2+2-2+9 = 11$

$y[2] = 11$

$n=3$

$y(3)$



$$y[3] = \sum_{k=1}^{3} x(n)\, h(3-k)$$

$= x(-1)\; h(-1) + x(0)\, h(0) + x(1)\, h(1) + x(2)\, h(2)$

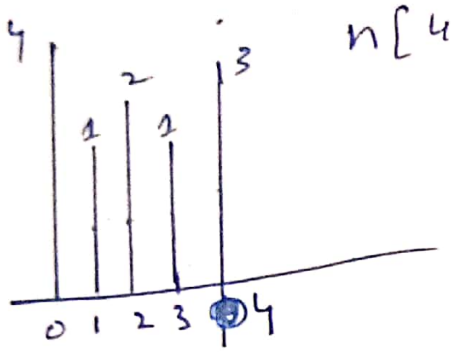
$= 2 \times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 4+1-4+3-12 = -8$

$y[3] = -8$

$n=4$

8)



$$y[4] = \sum_{k=0}^{3} x[n]\, h[4-k]$$

$$= x(0)\; h(0) + x(1)\, h(1) + x(2)\, h(2)$$

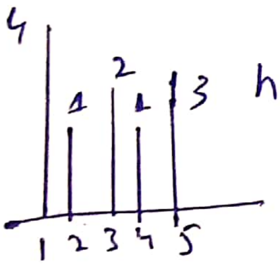
$$+ x(3)\, h(3)$$

$$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$

$$= 4-2+6-4 \quad = 10-6 = 4$$

$$y[4) = 4$$

$n=5$



$$y[5] = \sum_{k=1}^{3} x[n]\, h(5-k)$$

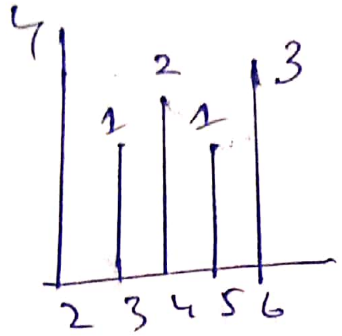
$$= x(1)\; h(1) + x(2)\, h(2) + x(3)\, h(3)$$

$$= (-2)(4) + 3(1) + (-4)(2)$$

$$-8+3-8 \quad = -13$$

$$y[5) = -13$$

$n = 6$



$h[6-k]$

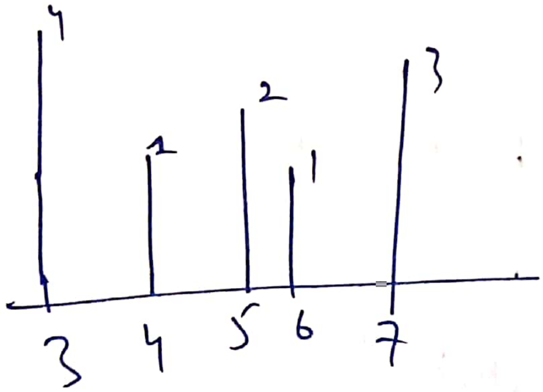
$$y[6] = \sum_{k=2}^{3} x(2)h(2) + x(3)\,h(3)$$

$$y[6) = (3)(4) + 1(-4)$$

$$y(6) = 12 - 4 = 8$$

$$y[6) = 8$$

$n = 7$



$$y[7] = x(3)\,h[3)$$

$$4 \times (-4)$$

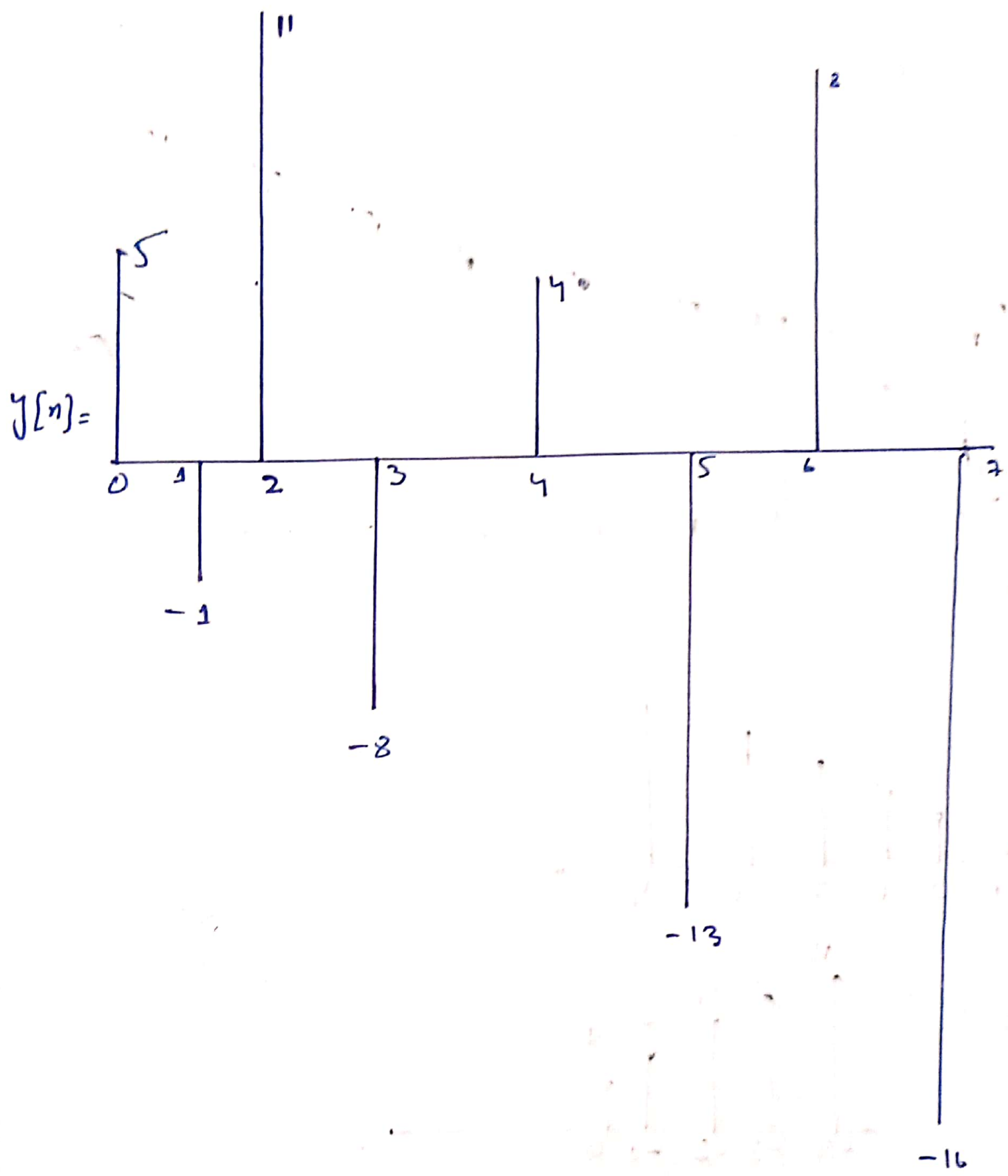
$$= 16$$

$$y(7) = -16 =$$

P.t.O

(10)

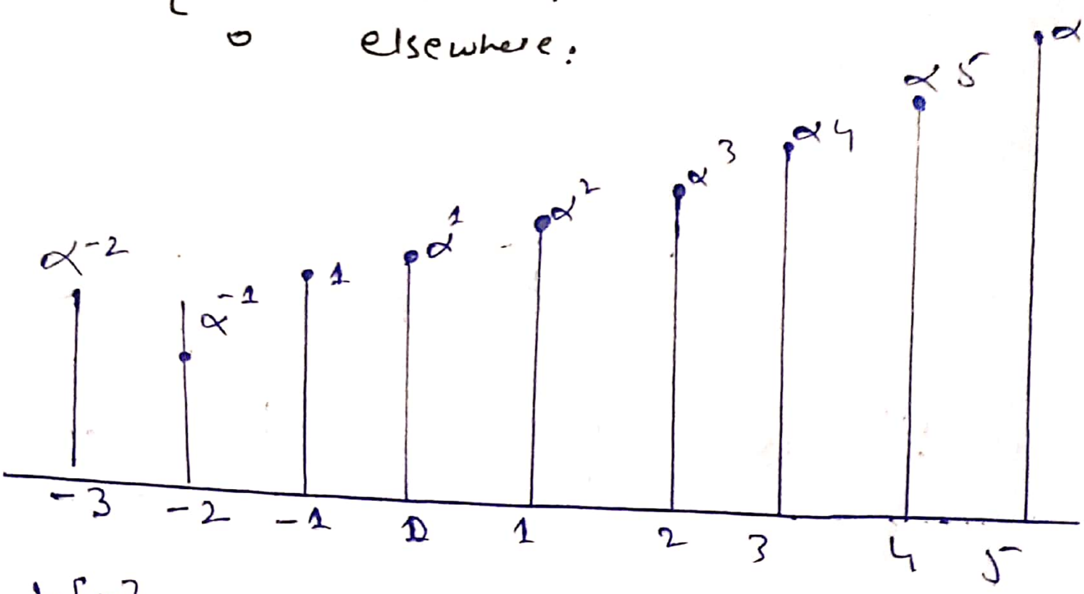
$y[n] =$



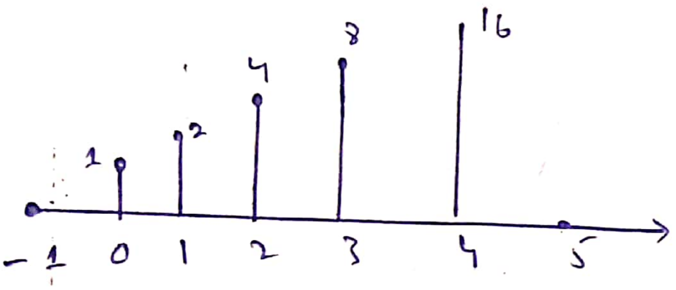
Q2: (b) Part:

$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

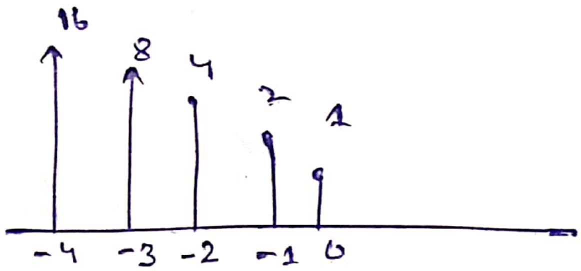
$$h[n] = \begin{cases} 2^n & 0 \le n < 4 \\ 0 & \text{elsewhere.} \end{cases}$$



h[n]



h[-n]



$y[-3] = 1 \times \alpha^{-2}$

$y[-2] = 1 \times \alpha^{-1} + 2 \times \alpha^{-2}$

$y[-1] = 1 \times 1 + 2 \times \alpha^{-1} + 4 \times \alpha^{-2}$

$y(0) = 1 \times \alpha^{1} + 2 \times 4 + 4 \times \alpha^{-1} + 8 \times \alpha^{-2}$

$y(1) = 1 \times \alpha^{2} + 2 \times \alpha^{1} + 4 \times 1 + 8 \times \alpha^{-1} + 16 \times \alpha^{-2}$

(12)

$y[2] = 1\times\alpha^{3} + 2\times\alpha^{2} + 4\times\alpha^{1} + 8\times 1 + 16\times\alpha^{-1}$

$y[3] = 1\times\alpha^{4} + 2\times\alpha^{3} + 4\times\alpha^{2} + 8\times\alpha^{1} + 16\times 1$

$y[4] = 1\times\alpha^{5} + 2\times\alpha^{4} + 4\times\alpha^{3} + 8\times\alpha^{2} + 16\times\alpha^{1}$

$y[5] = 1\times\alpha^{6} + 2\times\alpha^{5} + 4\times\alpha^{4} + 8\times\alpha^{3} + 16\times\alpha^{2}$

$y[6] = 2\times\alpha^{6} + 4\times\alpha^{5} + 8\times\alpha^{4} + 16\times\alpha^{3}$

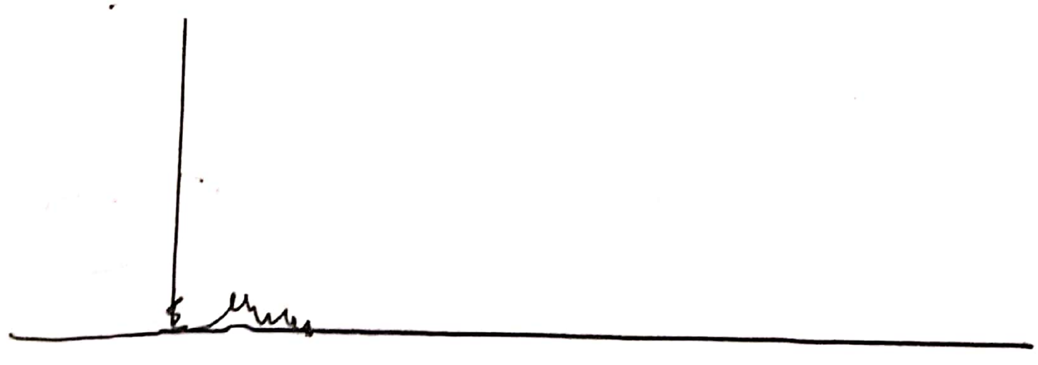
$y[7] = \overset{4\times\alpha^{6}}{\cancel{4\times\alpha^{6}}} + 8\times\alpha^{5} + \cancel{16\times\alpha^{4}}\ 16\times\alpha^{4}$

$y[8] = 8\times\alpha^{6} + 16\times\alpha^{5}$

$y[9] = 16\times\alpha^{6}$

$y[10] = 0$

Draw $y[n]$



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Q8: Part 1

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

Solution:

The Z-transform Pair is

$$x(n) = a^n u(n) \Rightarrow X(z) \frac{1}{1-\alpha z^{-1}} \quad \text{Roc}|z|$$

Put the value of above equation we get

$$x(z) = \sum_{n=0}^{\alpha} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\alpha} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

using geometric Series

$$\frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

taking lcm

$$\frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-(1-\frac{1}{3}z-\frac{1}{4}z^{-1}+\frac{1}{12}\cancel{z}\cancel{z})}{(1-\frac{1}{4}z^{-2})(1-\frac{1}{3}z)}$$

$$= \frac{1-\cancel{\frac{1}{3}z}+1-\cancel{\frac{1}{4}z^{-1}}-1+\cancel{\frac{1}{3}z}+\cancel{\frac{1}{4}z^{-1}}+\frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

P.T.O

Q3:

Part II

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3, & n \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Sollution

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Write in Z Transformm.

$$x(z) = \sum_{n=0}^{\alpha} \left(\tfrac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\alpha} 3^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

taking lcm:

$$\frac{1-3z^{-1} - 1 - \frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$= \frac{-\frac{5}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

The ROC is $|z|>3$