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Submmited to

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1 Orthogonal Matrix:

In linear algebra an is a square matrix whose ~~Solution~~ columns and rows are orthogonal unit vectors (orthonormal vectors). One way to express this is. Where is the transpose of Q and i the identity matrix.

(2) basis for a vector Space:

A vector space's that are linearly independent and span the Space. A basis is linearly independent because the vectors in it cannot be defined as a linear combination of any of the other vectors in the basis.

for example.

$$S = \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad V = R^3$$

③ The span of vector space:

The span of a set of vectors is the set of all all linear Combination of the vectors for example.

if and then the span of v^1 and v^2 is the set of all vectors of the form $sv^1 + tv^2$ for some scalars s and t. The span of a set of vector in.

The dimension of vector space.

Dimension (vector space) in Mathematics, the dimension of a vector space V is the cardinality (i.e. the number of vectors) of a basis of V over its base field. it is sometimes called Hamel dimension.

$$S = \left\{ \begin{bmatrix} 3a + 6b - c \\ 6a - 2b - 2c \\ -9a + 5b + 3c \\ -3a + b + c \end{bmatrix} \right\} a, b, c \text{ real}$$

eigenvector:

In linear algebra an ~~e~~ eigenvector or characteristic vector of a linear transformation, is a nonzero vector that changes at most by a scalar factor when that linear transformation is applied to it. The corresponding eigenvalue is the factor by which the eigenvector is scaled.

sub space vector:

In mathematics and more specifically in linear algebra, a linear subspace also known as a vector subspace, is a vector space that is a subset of some large vector space. A linear subspace is usually called simply a subspace when the context serves to distinguish it form other types of subspaces.

One to One Linear Transformation:

Recall that a function is 1-1 if $f(x) = f(y)$ implies that

$$x = y$$

Since a linear transformation is defined as a function the definition of 1-1 carries over to linear transformation.

A linear transformation L is 1-1 if for all vectors u and v

$$L(u) = L(v)$$

implies that

$$u = v$$

~~Rank~~ of linear transformation:
The nullity of T is the dimension of its kernel while the rank of T is the dimension of its image. These are denoted nullity T and rank respectively. Given coordinate systems for V and W, so that every linear transformation T can be described by a matrix A so that $T(x) = Ax$.

The image of a linear transformation:
or matrix is the span of the vectors of linear transformation (think of it as what vectors you can get from applying the linear transformation or multiplying the matrix by a vector) it can be written as $Im(A)$.

rank of a linear transformation
The rank of a linear transformation L is the dimension of its image written rank L. The

nullity of a linear transformation is the dimension of the Kernel written L theorem Dimension $V \rightarrow W$ be a linear transformation.

Characteristic Polynomial:

The characteristic Polynomial of a square matrix is a Polynomial which is invariant under Matrix similarity and has the eigenvalues as roots It has the the determinant and the trace of the Matrix as Coefficients.

The characteristic polynomial of an endomorphism of vector spaces of finite dimension is the characteristic polynomial the matrix of the endomorphism over any base it does not depend on the choice of a basis.

Equivalence relation:

in Mathematics an equivalence relation is a binary relation that is reflexive, symmetric and transitive. The releation is equal to is the canonical example of an equivalence relation where for any objects a,b and c a=a if a=b then b=a and if a=b and b=c then a=c.

linear homogenous equation:
A linear differential equation is homogenous if it is a homogenous linear equation in the unknown function and its derivatives it follow that if is a Solution So is , for any (non Zero) constant c.

General Solution ~~to a~~
A solution yP(x) of a differential equation that contains is called a Particular solution to the equation $a2(x)y'' + a1(x)y' + a0(x)y = r(x)$

System of linear equation:
system of linear equation is a collection of one or more linear equation involving the Same Set of

variables [1] [2] [3] [4] [5] for example

$$3x + 2y - z = 1$$
$$2x - 2y + 4z = -2$$
$$-x + \frac{1}{2}y - z = 0$$

Direct sums of subspace vector:
One such example of a direct sum comes from the vector space. Let $U_1 \{(x,0,0) : x \in R\}$ $U_2 \{(0,y,0) : y \in R\}$ and is unique determined since the first coordinate is determined only by vectors in V_3 therefore

orthogonal compenient
In the mathematics field of linear algebra and funcation ~~al~~ analysis the orthogonal complement of a subspace W of a vector space V equipped with a bilinear form B is the set $W^{\perp}$ of all vectors in V that are orthogonal to every vector i in W.

"Question No 2"

$$x + y + z + w = 1$$
$$x + 2y + 2z + 2z = 1$$
$$x + 2y + 3z + 3w = 1$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{V}$$

$$MX = V$$

$$X = M^{-1} V$$

$$M^{-1} = \frac{\operatorname{adj} M}{|M|}$$

$$\operatorname{adj} M = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$|M| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{vmatrix}$$

$R_2 - R_1 \quad R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \end{vmatrix}$$

$$1 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$1 \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$

$$1(2-2) - 1(2-1) + 1(2-1)$$

$$-1 + 1 = 0$$

So it rank is Zero

it can solve because

$\boxed{\text{Question No: 3}}$

$$\det\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Sol:

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

adj $4-6=-2$ Rank

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Sol:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$x+y+z+\ldots = 1$

$x+2y+2z+2z = 1$

$x+2y+3z+3w=1$

$$1\begin{vmatrix}5&6\\8&9\end{vmatrix} - 2\begin{vmatrix}4&6\\7&9\end{vmatrix} + 3\begin{pmatrix}4&5\\7&8\end{pmatrix}$$

$$1(45-48)-2(36-42)+3(32-35)$$

$$-3-2(-6)+3(-3)$$

$$-3+12-9 = 0 \text{ rank}$$

$$\det\begin{pmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\\13&14&15&16\end{pmatrix}$$

Sol:

$$1\begin{vmatrix}1&2&3&4\\5&6&7&8\\9&10&11&12\\13&14&15&16\end{vmatrix}$$

Question

Expr…

dec…

Que…

No…

$R_2 - 5R_1$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5-5 & 6-10 & 7-15 & 8-20 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix}$$

$R_3 - 9R_1,\ R_4 - 13R_1$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 10-18 & 11-27 & 12-36 \\ 0 & 14-26 & 15-33 & 16-52 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \\ 0 & -12 & -18 & -36 \end{vmatrix}$$

$$1 \begin{vmatrix} -4 & -8 & -12 \\ -8 & -16 & -24 \\ -12 & -18 & -36 \end{vmatrix}$$

$$-4 \begin{vmatrix} -16 & -24 \\ -18 & -36 \end{vmatrix} + 8 \begin{vmatrix} -8 & -24 \\ -12 & -36 \end{vmatrix} -12 \begin{vmatrix} -8 & -16 \\ -12 & -18 \end{vmatrix}$$

$$-4(576-432)+8(288-288)-12(192-144)$$

$$= -576 + 0 - 576$$

$$-1152$$

$$\det\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \end{vmatrix}$$

$R_2 - 6R_1 \quad R_3 - 11R_1 \quad R_4 - 16R_1$

$$\begin{array}{r} \\ 6 \\ 11 \\ 16 \end{array}\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 7-12 & 8-18 & 9-24 & 10-30 \\ 0 & 12-22 & 13-33 & 14-44 & 15-55 \\ 0 & 17-32 & 18-48 & 19-64 & 20-80 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & -10 & -20 & -20 & -40 \\ 0 & -15 & -20 & -45 & -60 \end{vmatrix}$$

he image of a linear trans...
he rank of a linear transformatio...
he characteristic polynomial of a...
n equivalence relation.
homogeneous solution to a line...
particular solution to a linear s...
he general solution to a linear s...
he direct sum of a pair of subsp...
he orthogonal complement to...

No. 2: Consider the system of...

his system as a matrix equati...
osition for the matrix M (be s...

No. 3: Compute the followi...

$\det\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ . ...

your skills on

Determinant

$$\begin{bmatrix} 1 & 2 & 3 & n \\ n+1 & n+2 & n+3 & 2n \\ 2n+1 & 2n+2 & 2n+3 & 3n \\ n^2-n+1 & n^2-n+2 & n^2-n+3 & n^2 \end{bmatrix}$$

Sol:

$$A = \begin{bmatrix} 1 & 2 & 3 & n \\ n+1 & n+2 & n+3 & 2n \\ 2n+1 & 2n+2 & 2n+3 & 3n \\ n^2-n+1 & n^2-n+2 & n^2-n+3 & n^2 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 3R_1$

$R_4^2 - (R_1)^2$

$$|A| = \begin{vmatrix} 1 & 2 & 3 & n \\ n-1 & n-2 & n-3 & 0 \\ 2n-2 & 2n-4 & 2n-6 & 0 \\ n^2-n & n^2-n-2 & n^2-n-6 & 0 \end{vmatrix}$$

$$|A| = \begin{vmatrix} n-1 & n-2 & n-3 \\ 2n-2 & 2n-\text{[illegible]} & \text{[illegible]} \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{vmatrix}$$

$= (n-1)\{(2n-4)(n^2-n-6) - (n^2-n-2)(2n-6)\}$

$- (n-2)\{(2n-2)(n^2-n-6) - (n^2-n)(2n-6)\}$

$+ (n-3)\{(2n-2)(n^2-n-2) - (n^2-n)(2n-4)\}$

$= n-1(2n^3-6n^2-8n+24) - (2n^3-8n^2+2n+12)$

$= -n-2(2n^3-4n^2-10n+12) - (2n^3-8n^2+6n)$

$+ n-3(2n^3-4n^2-2n+4) - (2n^3-6n^2+4n)$

$= n-1(2n^3-6n^2-8n+24-2n^3+8n^2-2n-12)$

$-n-2(2n^3-4n^2-10n+12-2n^3+8n^2-6n)$

$+(n-3)(2n^3-4n^2-2n+4-2n^3+6n^2-4n)$

$=(n-1)(2n^2-10n+12) - n-2(4n^2-16n+12)$

$+(n-3)(2n^2-6n+4)$

$= n^3-11n^2+22n-12 - 4n^3+24n^2-44n+24$

$+ 2n^3-12n^2+22n-12$

$\boxed{= -n^3+n^2}$ Ans