

"Mid Paper Summer 2020"

ID :- 7493

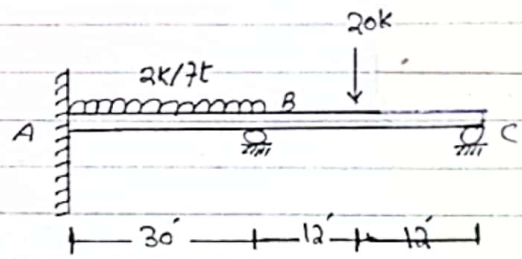
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Subject :- Structure Analysis - II

Submitted to :- Engr Adeed Khan

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Question #01



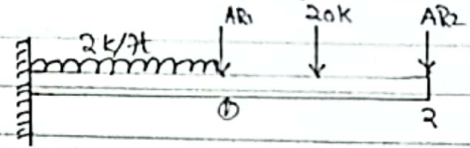
$EI = \text{Constant}$

Solution :-

Structural Indeterminacy = 2°

Step#1

Select Redundant Actions

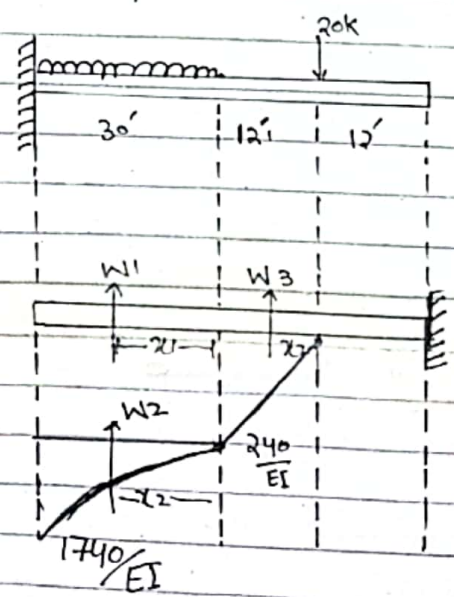


$$\begin{bmatrix} DRS1 \\ DRS2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} AR1 \\ AR2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$DRS = [DRL] + [F] \times [AR]$$

Step#2

Compute the value of [DRL]



$$20 \times 12 = 240$$

$$20 \times (12+30) + 2 \times 30 \times 5 = 1740$$

(2)

$$W_1 = 1500 \times 30 = 4500$$

$$W_2 = \frac{1}{3} \times 30 \times 240 = 2400$$

$$W_3 = \frac{1}{2} \times 12 \times 240 = 1440$$

$$x_1 = \frac{b}{a} = \frac{30}{2} = 15'$$

$$x_2 = \frac{3}{n+2} x_1 = \frac{3}{2+2} \times 30 = 22.5'$$

$$x_3 = \frac{2}{3} x_1 = 8'$$

Now finding DRL :

$$DRL_2 = W_1 \times (x_1 + 24) + W_2 \times (x_2 + 24) + W_3 \times (x_3 + 12)$$

$$= 45000(15 + 24) + 2400(22.5 + 24) + 1440(8 + 12)$$
$$= 1755000 + 111600 + 28800$$

$$DRL_2 = 1895400/EI$$

$$DRL_1 = W_1(x_1) + W_2(x_2)$$

$$= 45000(15) + 2400(22.5)$$

$$= 675000 + 54000$$

$$= 729000$$

So,

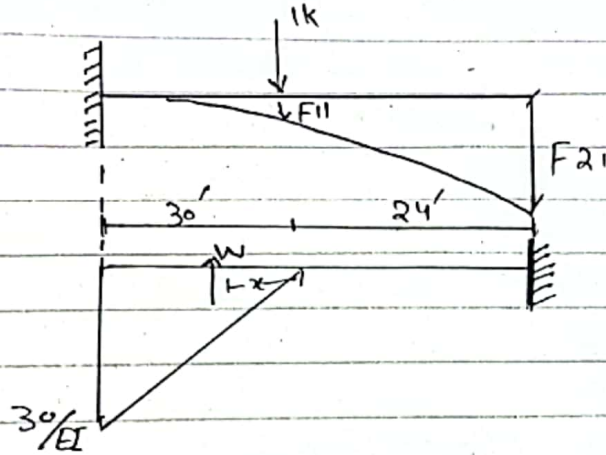
$$DRL_1 = \frac{1}{EI} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

Step#3 Flexibility Matrix

$$[F]_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

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(a) Applying Unit Load on AR_1



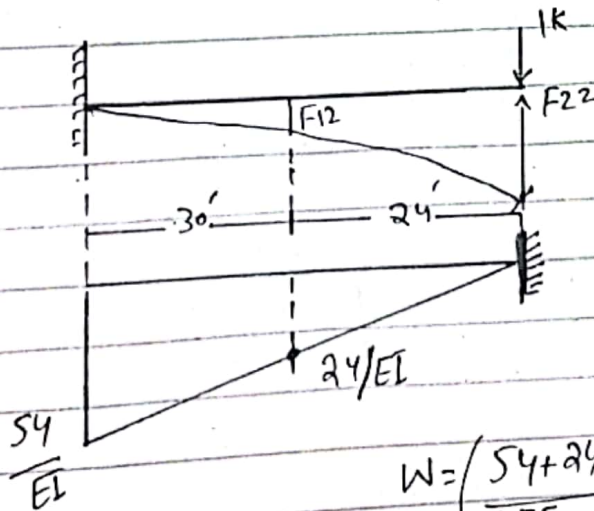
$$x = \frac{2}{3} \times 30 = 20'$$
$$W = \frac{1}{2} (30/EI \times 30)$$
$$= 450/EI$$

So,

$$F_{11} = \frac{450}{EI} (20) = 9000/EI$$

$$F_{21} = \frac{450}{EI} (20 + 24) = 19800/EI$$

Now apply unit load on AR_2



$$W = \left(\frac{54 + 24}{2EI} \right) \times 30$$
$$= 1170/EI$$

(4)

Now the distance

$$\begin{aligned}x &= \frac{L}{3} \left[\frac{b+2(a)}{a+b} \right] \\ &= \frac{30}{3} \left[\frac{24+2(54)}{54+24} \right] = 16.92'\end{aligned}$$

$$\Rightarrow F_{12} = \frac{1170}{EI} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92 + 24) = \frac{47876.4}{EI}$$

Hence

$$F_{2 \times 2} = \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} \frac{1}{EI}$$

Step #4 Compute the value of AR

$$[D_{RS}] = [D_{RL}] + [F] \times [AR]$$

$$[AR] = [D_{RS} - D_{RL}] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{|F|} \times \text{Adj } F$$

$$= \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{vmatrix}} \times \text{Adj} \begin{bmatrix} 19000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix}$$

(5)

$$|F| = (9000 \times 47876.4 - 19796.4 \times 19800) \\ (430887600 - 391968720)$$

$$|F| = 38918880$$

$$\Rightarrow \text{Ad}A = \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 & -729000 \\ 0 & -1895400 \end{bmatrix} \frac{1}{E} \times \frac{1}{38918880} \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \frac{1}{E} \times \frac{\begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}}{38918880}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$

⑤

Answer #2

Force Method	Displacement Method
→ It is also known as flexibility matrix method or compatibility method	→ It is called as equilibrium method or stiffness method matrix
→ In force method the unknown are taken as force or reaction	→ In displacement method the unknown are taken as joint displacement.
→ In this the number of redundant = D_s	→ In this the number of Redundant = D_k
→ In this the forces are found by compatibility equation of displacement	→ In this the displacement are found by equilibrium equation of force.
→ In this the type of indeterminacy is static indeterminacy	→ In this the type of indeterminacy is kinematic indeterminacy.
It is suitable when $D_s < D_k$	It is suitable when $D_s > D_k$

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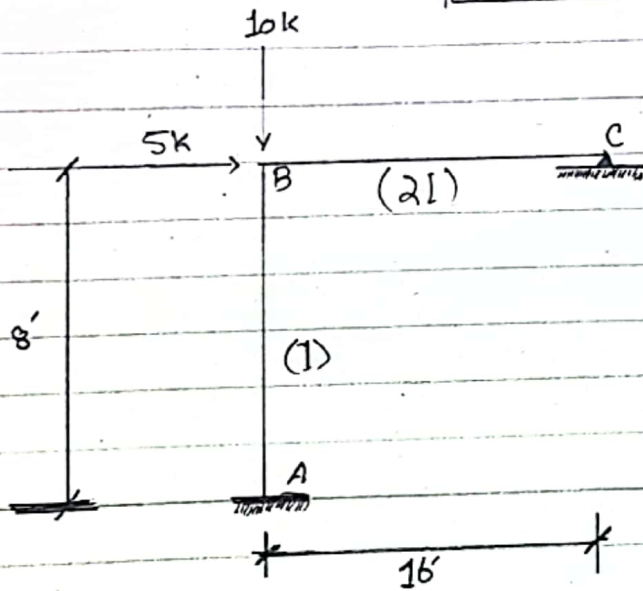
Suitable Method:

- (i) For analysis structure of matrix approach both the force method or displacement method can be used depend upon solution.
 - (ii) When the degree of static Indeterminacy (D_s) is less than the degree of kinematic Indeterminacy (D_k) example: $D_s < D_k$ than it is suggested to use force method of analysis.
 - (iii) when the degree of static Indeterminacy (D_s) is more than the degree of kinematic Indeterminacy (D_k), $D_k < D_s$. Then it is suggested to use the displacement method of Analysis.
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Question #03

Flexibility Method

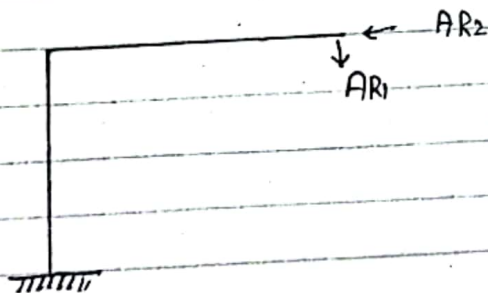


Solution:-

Total statical indeterminacy
 $\Rightarrow R-3 = 5-3 = 2$

Step #01

Identify Redundant Action



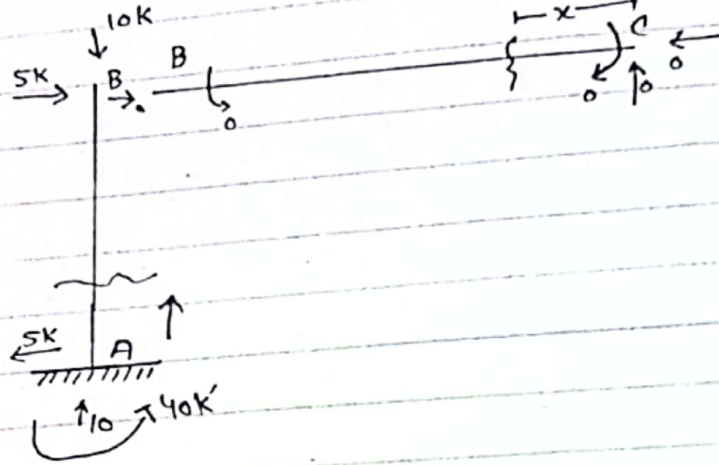
$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(9)

Step#02

Compute

value of [DRL]



Step#03

[F] or [AMR]

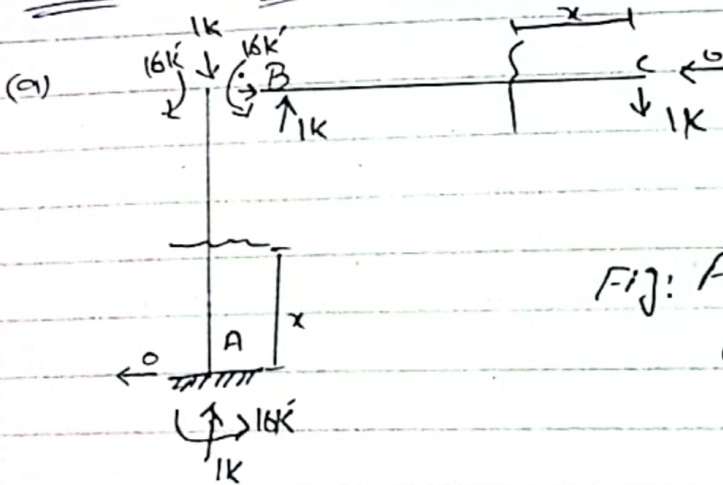


Fig: AMR-values
(m₁ values)

(b)

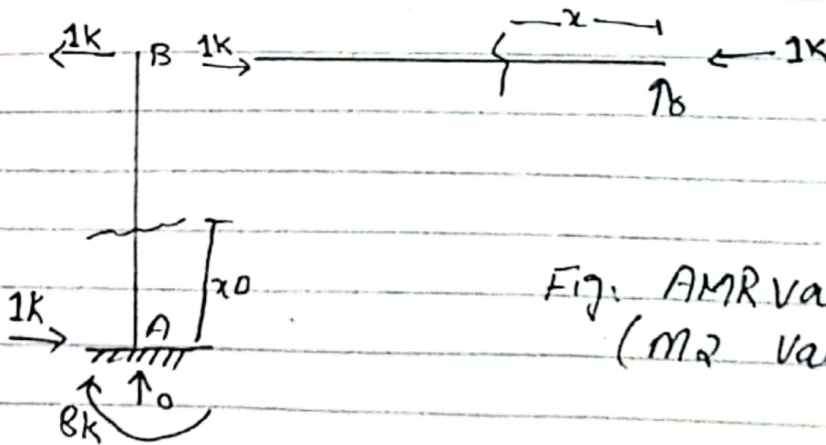


Fig: AMR values
(m₂ value)

(10)

Member	AB	BC
origin	A	C
Limits	0-8	0-16
I	I	2I
M	$5x-40$	0
m_1	-16	x
m_2	$8-x$	0

For Finding value of DRL

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot m_1(x)}{EI} dx + \int_0^{16} \frac{M_{BC} \cdot m_2(x)}{EI} dx$$

$$= \int_0^8 \frac{(5x-40)(-16)}{EI} dx + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx$$

$$DRL_1 = \frac{2560}{EI}$$

$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x)}{EI} dx + \int_0^{16} \frac{0 \cdot 0}{E(2I)} dx$$

$$DRL_2 = \frac{-953.33}{EI}$$

Compute Flexibility Matrix

$$F_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

(11)

$$\begin{aligned} \rightarrow F_{11} &= \int_0^8 \frac{m_1^2 (AB)}{EI} dx + \int_0^{16} \frac{m_1^2 (BC)}{EI} dx \\ &= \int_0^8 \frac{(-16)^2}{EI} dx + \int_0^{16} \frac{m_1^2 (BC)}{EI} dx \end{aligned}$$

$$F_{11} = \frac{2730.67}{EI}$$

$$\rightarrow F_{12} = F_{21} = \int_0^8 m_1(AB) \cdot m_2(AB) dx + \int_0^{16} m_1(BC) \cdot m_2(BC) dx$$

$$= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{E(I_{x2})} dx$$

$$F_{12} = F_{21} = \frac{-512}{EI}$$

$$\begin{aligned} \rightarrow F_{22} &= \int_0^8 (m_1)^2 AB dx + \int_0^{16} (m_2)^2 BC dx \\ &= \int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{E(I_{x2})} dx \end{aligned}$$

$$F_{22} = 170.67$$

As we know

$$[DRS] = [DRL] + [AR] \cdot [F]$$

$$[AR] = \frac{[DRS] - [DRL]}{[F]}$$

(12)

$$[AR] = [F]^{-1} \times [DRS - DRL]$$

$$= EI \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 & -2560 \\ 0 & +853.33 \end{bmatrix} \times \frac{1}{EI}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00025 \\ 4.97 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$