

Name → Mujahid Khan

ID → 14582

Subject → Operation Research

Section → (A)

Class → Monday Timing 8:00 = 11:00

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Q1: There are total of 5 machines and five employments are to be relegated and the cost network is as per the following. Locate the best possible task?

		MACHINES				
		A	B	C	D	E
J	1	6	12	3	11	15
O	2	4	2	7	1	16
B	3	18	11	10	7	11
S	4	16	19	122	23	21
	5	9	5	7	6	10

		Machines					
		1	2	3	4	5	Row minimum
<u>Solutions</u>	1	6	12	3	11	15	
Jobs	2	4	2	7	1	16	
	3	8	17	10	7	11	
	4	10	19	122	23	21	
	5	9	5	7	6	10	

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Row Reduction

Machines

	1	2	3	4	5
1	3	9	6	8	12
2	3	1	6	0	9
3	1	4	3	0	4
4	0	3	106	7	5
5	4	0	2	1	5
	0	0	0	0	4

Column reduction

	1	2	3	4	5
1	3	9	0	8	8
2	3	1	6	0	5
3	1	4	3	0	0
4	0	3	106	7	1
5	4	0	2	1	1

$S=5$ optimal solution

JOBs	MACHINES	TIME
1	3	3
2	4	1
3	5	11
4	1	10
5	2	5
		36

Total processing time = 36 km -

Q No (2)

Solve the following linear programming problem -

$$\text{min} = 2x_1 + 3x_2$$

$$\text{s.t.} - \left(\frac{1}{2}\right)x_1 + \left(\frac{1}{4}\right)x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1, x_2 \geq 0$$

→ * ————— * ← * →

Solution ⇒

Big M method for a max (min) linear programming problem.

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→ Introduce artificial variable in each row
(with no basic variable).

Put the artificial variable into the objective
function for max problem -

$$\max z = C^T x - M a_1 - M a_2 \dots M a_m -$$

(for minimum problem $\min z = C^T x + M a_1 + M a_2$
 $\dots + M a_m -$)

→ "clean up" the objective function -

→ solve the LP by simple -

If at optium all $a_i = 0$ we got the
optimal solution for the original LP
otherwise (some $a_i > 0$ at opt)

the original LP is infeasible -

$$\min z = 2x_1 + 3x_2$$

$$\text{s.t. } \left(\frac{1}{2}\right)x_1 + \left(\frac{1}{4}\right)x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1 + x_2 \geq 0$$

Standard form -

$$\min z = -2x_1 - 3x_2 = 0$$

$$\text{s.t. } \left(\frac{1}{2}\right)x_1 + \left(\frac{1}{4}\right)x_2 + s_1 = 4$$

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$$x_1 + 3x_2 - e_2 = 20$$

$$x_1 + x_2 \leq 10$$

$$x_1, x_2, s_1, e_2 \geq 0$$

⇒ Add artificial variable in constraints
2 and 3

$$\text{min } z = 2x_1 + 3x_2 - M a_2 - M a_3 = 0$$

$$\text{s.t. } (1/2)x_1 + (1/4)x_2 + s_1 = 4$$

$$x_1 + 3x_2 - e_2 + a_2 = 20$$

$$x_1 + x_2 + a_3 = 10$$

$$x_1, x_2, s_1, e_2, a_2, a_3 \geq 0$$

⇒ Tableau before "clean up"

z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	-2	-3	0	0	-M	-M	0
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

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→ First tableau (after "clean up") -

Z	x_1	x_2	s_1	e_2	a_2	RHS a_3	RHS
1	$2m-2$	$4m-3$	6	-m	0	0	$30m$
0	$1/2$	$1/4$	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

→ x_2 enters a_2 leaves the basis

Next tableau -

Z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	$(2m-3)/3$	0	0	$(m-3)/3$	$(3-4m)/3$	0	$(60+10m)/3$
0	$5/12$	0	-1	$1/12$	$-1/12$	0	$7/3$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3$

→ x_1 enters a_3 leaves the basis - next tableau -

Z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	0	0	0	$-1/2$	$(1-2m)/2$	$(3-2m)/2$	25
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

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Q no 3 \Rightarrow use Vogel's Approximation method to obtain the initial feasible solution of -

Destination -

Origin	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Solutions

	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

Demand = Supply -

Balance transportation problem -

	1	2	3	4				
1	X	40	X	80	08			
	20	22	17	4	120	13	(13)	-
2	10	X	30	30	016			
	24	37	9	7	46 76	2	2	2 (17)
3	50	X	X	X				
	32	37	20	15	50	5	5	5 17
	66	46	36	16				
	50	0	0	300				
	4	(15)	8	3				
	4	-	8	3				
	8	-	(11)	8				
	8	-	-	8				

$$880 + 320 + 240 + 270 + 210 + (40 \times 22) + (80 \times 4) + (10 \times 24) + (30 \times 9) + (30 \times 7) + 1600$$

$$(50 \times 32) F \quad \boxed{3520}$$