Course: Discrete Structure
Program: BS (SE)
Instructor: Muhammad Abrar Khan
Examination: Final paper
Total Marks: 50
Date: June. 25, 2020

Note: Attempt all questions. Use examples and diagrams where necessary.

Q.1

a) Explain the concept of Biconditional statement.

ANS: **Biconditional statement**

If p and q are two statements then "p if and only if q" is a compound statement, denote as $p \leftrightarrow q$ and referred as a biconditional statement or an equivalence. The equivalence $p \leftrightarrow q$ is true only when both p and q are true or when both p and q are false.

b) Let p, q, and r represent the following statements:

p: Sam had pizza last night.
q: Chris finished her homework.
r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.
ii. Pat watched the news this morning iff Sam did not have pizza last night.
iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.
iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

i)Sam had pizza last night if and only if Chris finished her homework.
ANS: $p \Leftrightarrow q$

ii)Pat watched the news this morning iff Sam did not have pizza last night.
ANS: $r \Leftrightarrow \neg p$

iii) Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.
ANS: $r \Leftrightarrow (q \land \neg p)$

iv) In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.
ANS: $r \Leftrightarrow (p \land q)$

Q.2

a) Lets p, q, r represent the following statements:

p: it is hot today.

q: it is sunny

r: it is raining

Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$

ii. $p \leftrightarrow (q \wedge r)$

iii. $p \leftrightarrow (q \vee r)$

iv. $r \leftrightarrow (p \vee q)$

i)it is sunny if only if it is hot today.

ii)it is hot today if only if it is sunny and it is raining.

iii)it is hot today,it is sunny or it is raining.

iv)it is raining,it is hot today or it is sunny.

Q.3

a) Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments. (Note: Examples and truth table should not belongs to your book or slides)

ANS:**Argument:**

We have already encountered a few basic concepts related to propositions. They include true (*T*), false (*F*), tautology, contradiction, and $(\wedge)$, or $(\vee)$, not $(\sim)$ as well as implication $(\rightarrow)$.If we further define an equivalence connective $\leftrightarrow$ for any two propositions *p* and *q* by $(p \rightarrow q) \wedge (q \rightarrow p)$, denoted by $p \leftrightarrow q$, then the usual "order of precedence" is

1.connectives within parentheses, innermost parentheses first

2.$\sim$

3.$\wedge$, $\vee$

4.$\rightarrow$

5.$\leftrightarrow$

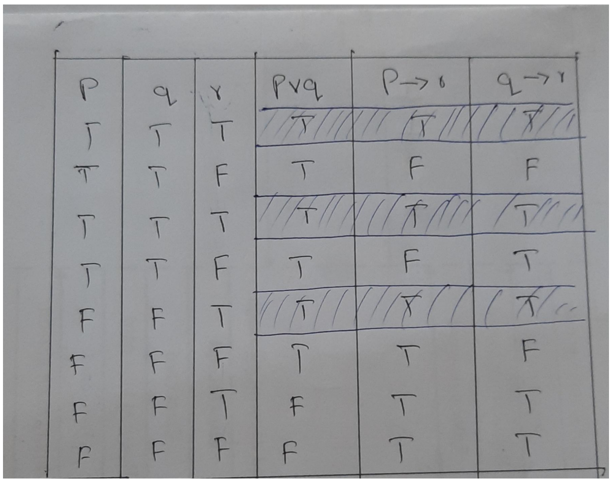
We note that $\wedge$ and $\vee$ are both left associative. From time to time you may also find that some people actually place a higher precedence for $\wedge$ than for $\vee$. To avoid possible confusion we shall always insert the parentheses at the appropriate places.

Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments.

Valid argument: an argument is valid if and only if it is necessary that if all of the premises are true, then the conclusion is true; if all the premises are true, then the conclusion must be true; it is impossible that all the premises are true and the conclusion is false.

For Example:

Solution From the table



p	q	r	p∨q	p→r	q→r
T	T	T	T	T	T
T	T	F	T	F	F
T	T	T	T	T	T
T	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

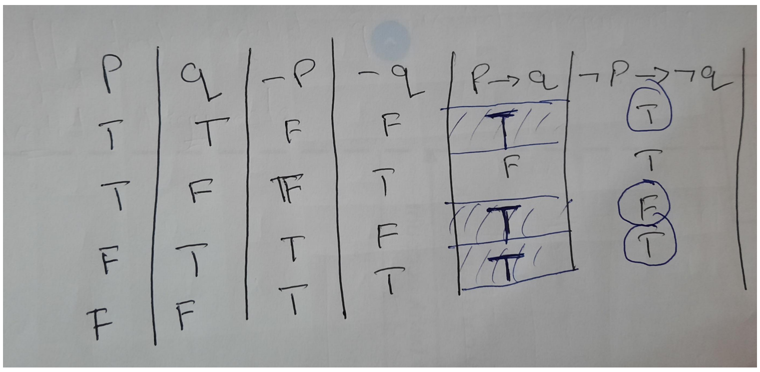
we see all critical rows (in this case, those with the shaded positions *all* containing a *T*) correspond to (the circled) *T*(true) for *r*. Hence the argument is valid.

<u>Invalid argument:</u> an argument that is not valid. We can test for invalidity by assuming that all the premises are true and seeing whether it is still possible for the conclusion to be false. If this is possible, the argument is invalid.

<u>For Example:</u>

Show that the argument ($p \rightarrow q, \therefore \sim p \rightarrow \sim q$) is invalid.

Solution



p	q	¬p	¬q	p→q	¬p→¬q
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

We see that on the 3rd row, a critical row, the premise $p \rightarrow q$ is true while the conclusion $\sim$ p $\rightarrow \sim$ q is false. Hence the argument ($p \rightarrow q, \therefore \sim p \rightarrow \sim q$) is invalid.

Q.4

a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

ANS: **Union:**Union of two given sets is the smallest set which contains all the elements of both the sets.To find the union of two given sets A and B is a set which consists of all the elements of A and all the elements of B such that no element is repeated.The symbol for denoting union of sets is '∪'.

We combine sets in much the same way that we combined propositions. Asking if an element x is in the resulting set is like asking if a proposition is true. ... Since the columns corresponding to the two sets match, they are equal.

For Example

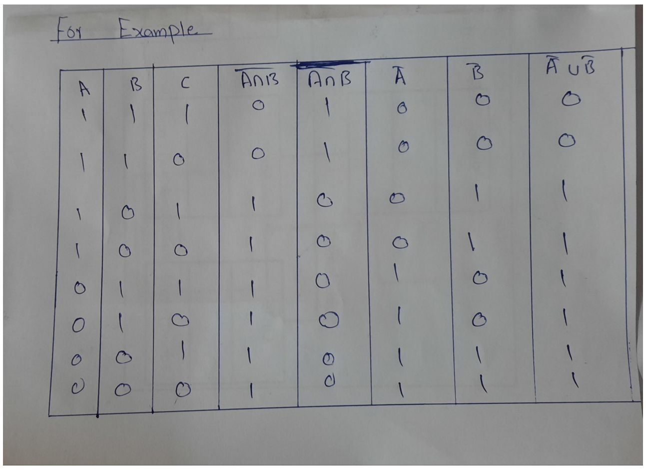
A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table. (Note: Examples and truth table should not belongs to your book or slides)

ANS:Intersection:

Intersection of two given sets is the largest set which contains all the elements that are common to both the sets.

To find the intersection of two given sets A and B is a set which consists of all the elements which are common to both A and B.The symbol for denoting intersection of sets is '∩'.



For Example

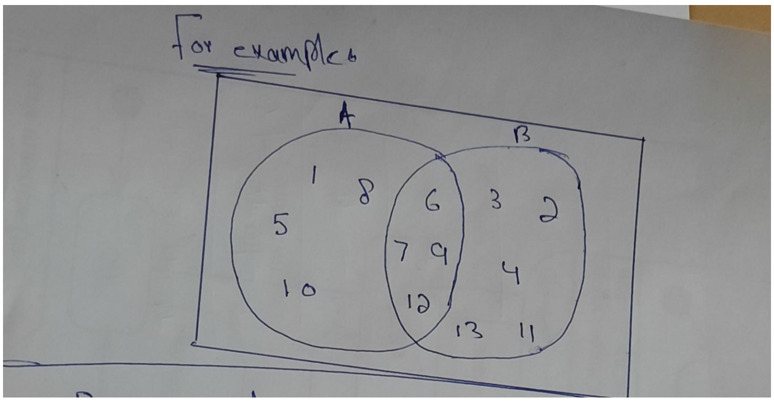
A	B	C	$\overline{A\cap B}$	$A\cap B$	$\bar{A}$	$\bar{B}$	$\bar{A}\cup\bar{B}$
1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	0
1	0	1	1	0	0	1	1
1	0	0	1	0	0	1	1
0	1	1	1	0	1	0	1
0	1	0	1	0	1	0	1
0	0	1	1	0	1	1	1
0	0	0	1	0	1	1	1

Q.5

a) Explain the concept of Venn diagram with examples.

ANS:<u>**Venn diagram:**</u> The term Venn diagram is not foreign since we all have had Mathematics, especially Probability and Algebra. Now, for a layman, the Venn diagram is a pictorial exhibition of all possible real relations between a collection of varying sets of items. It is made up of several overlapping circles or oval shapes, with each representing a single set or item.

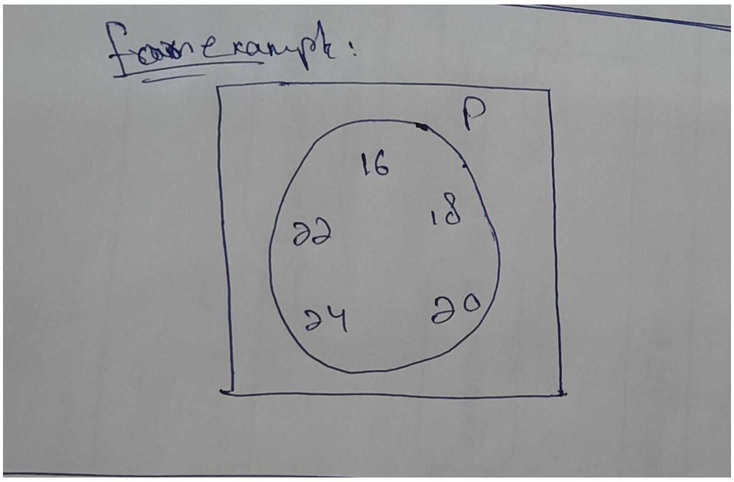
Venn diagrams depict complex and theoretical relationships and ideas for a better and easier understanding. These diagrams are also professionally utilized to display complex mathematical concepts by professors, classification in science, and develop sales strategies in the business industry.



b) Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.

ANS: List out the elements of *P*.

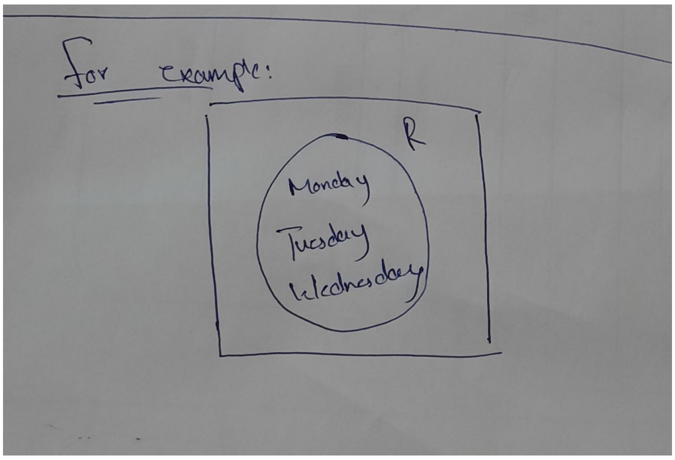
P = {16, 18, 20, 22, 24} ← 'between' does not include 15 and 25



c) Draw and label a Venn diagram to represent the set

***R* = {Monday, Tuesday, Wednesday}.**

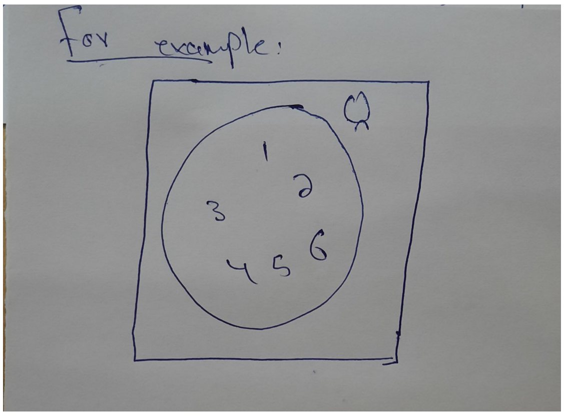
ANS: Draw a circle or oval. Label it *R* . Put the elements in *R*.



c) **Given the set $Q = \{x : 2x - 3 < 11, x$ is a positive integer $\}$. Draw and label a Venn diagram to represent the set Q.**

d) **ANS:** Since an equation is given, we need to first solve for x.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$



.So, $Q = \{1, 2, 3, 4, 5, 6\}$

Draw a circle or oval. Label it Q .

Put the elements in Q