

Name

Shohab malook

id

7878

Section

A

Subject

differential
equation.

Q1 ①
No 1

Find the Fourier Series representation of $f(t) = 1+t, -\pi \leq t \leq \pi$

Solution

$$f(t) = 1+t \quad -\pi \leq t \leq \pi$$

Here we use the formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \quad \text{eq } \textcircled{1}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t) dt$$

$$a_0 = \frac{1}{2\pi} \left[t + \frac{t^2}{2} \right]_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$a_0 = \frac{1}{2\pi} (2\pi + \pi^2)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \cos nt \, dt$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \frac{\sin nt}{n} dt (1+t) \right)$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{-1}{n+\pi} (\cos n\pi - \cos n(-\pi))$$

$$a_n = \frac{-1}{n+\pi} (-1 - (-1))$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt \, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin nt - \int \left(\sin nt \frac{d}{dt} (1+t) dt \right)$$

$$b_n = \frac{1}{\pi} \left((1+t) \frac{(-\cos nt)}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\cos nt}{n} (1) \right) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{-(1+t) \cos nt}{n} \Big|_{-\pi}^{\pi} + \left(\frac{\sin nt}{n^2} \Big|_{-\pi}^{\pi} \right) \right)$$

$$b_n = \frac{-1}{n\pi} \left((1+\pi) \cos n\pi \right) - \left((1+\pi) (\cos \pi) \right)$$

$$b_0 = \frac{-1}{n\pi} (2\pi \cos n\pi)$$

PL (3)

$$\text{Here } \cos n\bar{\pi} = \frac{(-1)^{n+1}}{n}$$

$$b_n = \frac{2}{n} (-1)^{n+1}$$

So equation become.

$$f(x) = \frac{1}{2n} (2\bar{\pi} + \bar{\pi}^2) + 0 \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t$$

Q 2 :: 20¹¹

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Sol.:

Step = 01

~~but~~ we have

$$(A - \lambda I) x = 0 \quad A = \text{Given matrix.}$$

Step = 02

we have the characteristic equation is given by.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step :: 03

Q2 (2)

13 - |Sum of Diagonal elem| λ^2 + |Sum of Diagonal matrix| $\lambda - |A| = 0$

Sum of Diagonal element = $1 + 1 + 2 = 4$

Sum of Diagonal matrix = $\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$$= (-6) + (2) + (1)$$

$$= -6 + 2 + 1$$

$$= -3$$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{--- (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2 - 8) - 0 + 1(6 - 0)$$

$$= -6 + 6$$

$$= 0$$

By putting value in (B).

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

using quadratic formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} a &= 1 \\ b &= -4 \\ c &= -3 \end{aligned}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigenvalues

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2} \right)$$

Ans:.

Q No 3

Q3 ①

Solve the following system of linear equation.

$$5x + 0 + 4z + 2m = 3$$

$$x - y + 2z + m = 1$$

$$4x + y + 2z + 0 = 1$$

$$x + y + 2z + m = 0$$

Sol:-

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right] \xrightarrow{R_4 \ R_2}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 + \frac{4}{5} & 1 \\ 0 & -1 & +\frac{6}{5} & +\frac{4}{5} & \frac{3}{5} \\ 0 & 2 & -1 & 0 & -1 \end{array} \right] \xrightarrow{-\frac{1}{5} \times R_2}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & \frac{6}{5} & \frac{4}{5} & \frac{3}{5} \\ 0 & 0 & \frac{7}{5} & \frac{8}{5} & \frac{1}{5} \end{array} \right] \xrightarrow{5 \times R_2 \text{ and } 5 \times R_4}$$

Q3 (2)

$$\left[\begin{array}{cccc|c} 5 & 1 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad \begin{array}{l} 5R_3 \text{ and } 5R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad \begin{array}{l} 1/5 \times R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad \begin{array}{l} R_2 \times 5 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array} \right] \quad \begin{array}{l} R_3 - R_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \quad \begin{array}{l} R_3 \rightarrow R_4 \\ 1/7 \times R_3 \\ 1/2 \times R_4 \end{array}$$

Q₃ (2) (3)

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \cdot (2x-5)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array} \right] \cdot 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 12/34 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array} \right]$$

$$\begin{array}{c} \text{Q3} \\ \text{Q4} \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 3 1/2 \\ 0 & 0 & 1 & 0 & -11/2 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 3 1/2 \\ 0 & 0 & 1 & 0 & -11/2 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right]$$

$$(x, y, z, m) = \left(\frac{3}{4}, 3 \frac{1}{2}, -\frac{11}{2}, \frac{1}{3} \right)$$

$$x = \frac{3}{4}$$

$$y = 3 \frac{1}{2}$$

$$z = -\frac{11}{2}$$

$$m = \frac{1}{3}$$

Q No 4

verify that

$$u(x, t) = \sin(x + 2t)$$

is a solution of the one-dimensional equation

Solution:-

Given that

$$u(x, t) = \sin(x + 2t)$$

Differentiate wr.t x partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x + 2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t) \frac{\partial}{\partial x} (x + 2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t) (1 + 0)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x + 2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \frac{\partial}{\partial x} (x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) (1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t 't'

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t) (0+2)$$

$$\frac{\partial u}{\partial t} = 2 \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t) (0+2)$$

$$\frac{d^2 u}{dt^2} = -4 \sin(x+2t)$$

we know that one dimensional wave equation is

$$\frac{d^2 u}{dt^2} = c^2 \frac{d^2 u}{dx^2}$$

$$-4 \sin(x+2t) = c^2 [-\sin(x+2t)]$$

$$-4 \sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4 \sin(x+2t) + c^2 \sin(x+2t) = 0$$

for the arbitrary constant $c = \pm 2$

$$-4 \sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4 \sin(x+2t) + 4 \sin(x+2t) = 0$$

$$0 = 0$$

then it will be verified
for the arbitrary constant
 $c = 2$.