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4th

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Electromagnetic field theory

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Q: 1 The value of E at $P(\rho = 2, \phi = 40^\circ, z = 3)$ is given as $E = 100 a_\rho - 200 a_\phi + 300 a_z$ V/m. Determine the incremental work required to move a $20 \mu\text{C}$ charge a distance of $6 \mu\text{m}$.

Ans in the direction of a_ρ : The incremental work is given by $dW = -qE \cdot dL$, where in this case, $dL = d\rho a_\rho = 6 \times 10^{-6} a_\rho$. Thus

$$dW = -(20 \times 10^{-6} \text{C})(100 \text{V/m})(6 \times 10^{-6} \text{m}) = -12 \times 10^{-9} \text{J} = -12 \text{nJ}$$

* in the direction of a_ϕ : in this case $dL = 2d\phi a_\phi = 6 \times 10^{-6} a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6}) = 2.4 \times 10^{-8} \text{J} = \underline{24 \text{nJ}}$$

* in the direction of a_z : Here, $dL = dz a_z = 6 \times 10^{-6} a_z$,
 $dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6}) = -3.6 \times 10^{-8} \text{J} = \underline{-36 \text{nJ}}$

* in the direction of E : Here, $dL = 6 \times 10^{-6} a_E$, where

$$a_E = \frac{100 a_\rho - 200 a_\phi + 300 a_z}{\sqrt{100^2 + 200^2 + 300^2}} = 0.267 a_\rho - 0.535 a_\phi + 0.802 a_z$$

Thus

$$dW = -(20 \times 10^{-6})[100 a_\rho - 200 a_\phi + 300 a_z] \cdot [0.267 a_\rho - 0.535 a_\phi + 0.802 a_z] \\ (6 \times 10^{-6}) = \underline{-44.9 \text{nJ}}$$

⇒ in the direction of $E = 2a_x - 3a_y + 4a_z$: in this case, $dl = 6 \times 10^{-6} a_E$, where

$$a_E = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z$$

So now

$$\begin{aligned} dw &= -(20 \times 10^{-6}) [100a_\rho - 200a_\phi + 300a_z] \cdot [0.371a_x - 0.557a_y + 0.743a_z] (6 \times 10^{-6}) \\ &= -(20 \times 10^{-6}) [37.1(a_\rho \cdot a_x) - 55.7(a_\rho \cdot a_y) - 74.2(a_\phi \cdot a_x) + 111.4(a_\phi \cdot a_y) + 222.9] (6 \times 10^{-6}) \end{aligned}$$

Where, at P, $(a_\rho \cdot a_x) = (a_\phi \cdot a_y) = \cos(40^\circ) = 0.766$,

$(a_\rho \cdot a_y) = \sin(40^\circ) = 0.643$, and $(a_\phi \cdot a_x) = -\sin(40^\circ) = -0.643$. Substituting these results in

$$\begin{aligned} dw &= -(20 \times 10^{-6}) [28.4 - 35.8 + 47.7 + 85.3 + 222.9] \\ (6 \times 10^{-6}) &= \underline{\underline{-41.8 \mu\text{J}}} \end{aligned}$$

Q2 Let $E = 400\hat{x} - 300\hat{y} + 500\hat{z}$ in the neighborhood of point $P(6, 2, -3)$. Find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by:

$\Rightarrow \hat{x} + \hat{y} + \hat{z}$: we write

$$dW = -qE \cdot dL = -4(400\hat{x} - 300\hat{y} + 500\hat{z}) \cdot \frac{(\hat{x} + \hat{y} + \hat{z}) (10^{-3})}{\sqrt{3}}$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{3}} (400 - 300 + 500) = -1.39\text{J}$$

$\Rightarrow -2\hat{x} + 3\hat{y} - \frac{3}{2}\hat{z}$: The computation is similar to that of part a, but we change the direction.

$$dW = -qE \cdot dL = -4(400\hat{x} - 300\hat{y} + 500\hat{z}) \cdot \frac{(-2\hat{x} + 3\hat{y} - \frac{3}{2}\hat{z}) (10^{-3})}{\sqrt{14}}$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = \underline{2.35\text{J}}$$

Q3 if $E = 120 \hat{a}_P$ V/m, Find The incremental amount of work done in moving a $50 \mu\text{M}$ charge a distance of 2 mm from.

Ans $P(1, 2, 3)$ Toward $Q(2, 1, 4)$: The vector along this direction will be $Q - P = (1, -1, 1)$ from which $\hat{a}_{PQ} = \left(\frac{a_x - a_y + a_z}{\sqrt{3}} \right)$. We now write

$$dW = -qE \cdot dL = - (50 \times 10^{-6}) \left[120 \hat{a}_P \cdot \left(\frac{a_x - a_y + a_z}{\sqrt{3}} \right) (2 \times 10^{-3}) \right]$$

$$= - (50 \times 10^{-6}) (120) \left[(\hat{a}_P \cdot a_x) - \hat{a}_P \cdot a_y \right] \frac{1}{\sqrt{3}} (2 \times 10^{-3})$$

At P , $\phi = \tan^{-1}(2/1) = 63.4^\circ$. Thus $(\hat{a}_P \cdot \frac{a}{\sqrt{2}}) = \cos(63.4) = 0.447$ and $(\hat{a}_P \cdot \hat{a}_y) = \sin(63.4) = 0.894$.

Substituting these, we obtain $dW = 3.1 \mu\text{J}$.

$\rightarrow Q(2, 1, 4)$ Toward $P(1, 2, 3)$: A little thought is in order here: Note that the field has only a radial component and does not depend on ϕ or z . Note also that P and Q are at the same radius ($\sqrt{5}$) from the z -axis, but have different ϕ and z coordinates. We could just as well position the two points at the same z location and the problem would not change.

if this were so, then moving along a straight

line between P & Q would involve moving along a chord of a circle with radius $\sqrt{5}$. Halfway along this line point of symmetry in field (make a sketch to see this).

This means that when starting from either point the initial force will be same. Thus the answer $dW = 31$
Hj

as part a. This is also found by going through the same procedure as part a, but with the direction (role of P & Q) reversed.

Q4 Compute the value of G using the path.

Ans: Straight line of segments A (1, -1, 2) to B (1, 1, 2) to P (2, 1, 2) in general

We have

$$\int_A^P G \cdot dL = \int_A^P 2y dx$$

The change of x occurs when moving b/w B and P during which $y = 1$. Thus

$$\int_A^P G \cdot dL = \int_B^P 2y dx = \int_1^2 2(1) dx = \boxed{2}$$

(B) Straight line segment A (1, -1, 2) to C (2, -1, 2) to P (2, 1, 2) in case the change in x occurs when moving from A to C, during

which $y = -1$. Thus

$$\int_A^P G \cdot dL = \int_A^C 2y dx = \int_1^2 2(-1) dx = \boxed{-2}$$

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QUESTION # 7.

A balance Δ -load to a 60Hz three phase ----- Phase draw 5KW.

(a) determine the load impedance

(b) Find I_a I_b I_c

a) STEP # 1.

$$|V_{\text{line}}| = \sqrt{3}V_p = 240 \rightarrow V_p = \frac{240}{\sqrt{3}} = 138.56$$

$$V = V_p \angle -30^\circ$$

$$\text{P.f} = 0.5 = \cos \phi$$

$$P = S \cos \phi \rightarrow S = \frac{P}{\cos \phi} = \frac{5}{0.5} = 10 \text{ KVA}$$

$$Q = S \sin \phi = 10 \sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ KVA}$$

But

$$S_p = \frac{V_p^2}{Z_p} \rightarrow Z_p = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66) \times 10^3}$$

QUESTION # 7.

A balance Y-load to a 60Hz three phase ----- Phase draw 5KW.

(a) determine the load impedance

(b) Find I_a I_b I_c

a) STEP # 1.

$$|V_{lab}| = \sqrt{3}V_p = 240 \rightarrow V_p = \frac{240}{\sqrt{3}} = 138.56$$

$$V = V_p \angle -30^\circ$$

$$P_f = 0.5 = \cos \theta$$

$$P = S \cos \theta \rightarrow S = \frac{P}{\cos \theta} = \frac{5}{0.5} = 10 \text{ KVA}$$

$$Q = S \sin \theta = 10 \sin 60 = 8.66$$

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