Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

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<table>
<tr><td rowspan="2">Q1.</td><td>(a)</td><td>Consider the following analog signal

$$x_a(t) = 3cos100\pi t + 4sin200\pi t$$
i. Determine the minimum sampling rate required to avoid aliasing.
ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.
iii. What is the analog signal $y(t)$ we can reconstruct from the samples if we use ideal interpolation?</td><td>Marks 5
CLO 1</td></tr>
<tr><td>(b)</td><td>Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$
This is signal is sampled at the rate $F_s = 2Hz$.

i. Draw the sampled signal.
ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .
iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.</td><td>Marks 5
CLO 1</td></tr>
<tr><td></td><td>(a)</td><td></td><td>Marks 5</td></tr>
</table>

Q2.		Determine the response of the system to the following input signal with given impulse response $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	CLO 2
	(b)	Compute the convolution y(n) of the following signal $x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$ $h(n) = \begin{cases} 2^{n}, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{3})^{n}, & n \geq 0 \\ (\frac{1}{4})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \geq 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

①

(1)

Q1

a) consider the following analog signals.

$$x_a(t) = \cancel{3\cos}\ 3\cos 100\pi t + 4\sin 200\pi t$$

i) Determine minimum sampling rate required to avoid aliasing.

$$f_s \geq 2 f_{max} \qquad f = \frac{\omega}{2\pi}$$

$$f_1 = \frac{100\pi}{2\pi} \qquad f_2 = \frac{200\pi}{2\pi}$$

$$f_1 = 50\,Hz \qquad f_2 = 100\,Hz$$

So f_2 is max (greater than f_1)

$f_s \geq 2 \times 100\,Hz$ sample frequency to avoid aliasing.

ii)

$$F_s = 100\,Hz$$

f_1 becomes

$$f_1' = \frac{f_1}{F_s} = \frac{\cancel{10}50}{\cancel{10}100} = 0.5\,Hz.$$

f_2 becomes

$$f_2 = \frac{f_2}{100} = \frac{100}{100} = 1\,Hz.$$

So $\omega_1' = 2\pi f_1'$ $\qquad$ $\omega_2' = 2\pi f_2'$

$\omega_1' = 2\pi \times 0.5$ $\qquad$ $\omega_2' = 2\pi \times 1$

$\omega_1' = \pi$ $\qquad$ $\omega_2' = 2\pi.$

(a)

2

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The signals are.

$x[n] = 3\cos \pi n + 4\sin 2\pi n.$

The Effect of sampling rate on the newely generated discrete time signals is that there will be no aliasing Phenomenon. means there will not present unwanted component in the reconstruction of the signals. The reconstruct orignal signals:

$\omega_1 = 100\pi \quad f_2 = 200\pi$

$f_1 = \frac{\cancel{100\pi}}{\cancel{2\pi}} \quad f_2 = \cancel{200\pi}$

$\boxed{f_1 = 50}$

(iii) What is the analog signal $y_a(t)$ we can reconstructed from the sampling if we use ideal interpolation,

Sol Folding frequency of the sampled signal is:

folding frequency $= F_s/2 \Rightarrow \frac{100}{2}$

$= 50\ Hz$

we have frequency of the orignal signals

$f_1 = 50\ Hz,\ f_2 = 100\ Hz$

Both the frequency are either equal or greater the folding frequency.

Hence for ideal interpolation we can construct the orignal signal.

$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t.$

P-T-O

③ (3)

The orignal signal is constructed because we use sampling frequency at Nyquist rate.
we can also reconstruct the signal for sampling frequency above the Nyquist rate.

Q1 ⓑ Consider a discrete time signal which is given by

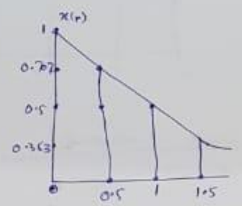
$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$F_s = 2Hz$$

$$F_s = 1/T \Rightarrow T = 1/F_s = \frac{1}{2} = 0.5 \text{ sec}$$

① Draw the sampled signal.

x_n	$= 0.5^n$
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 sec

ii P-TO

(4)

ii)

Sol:

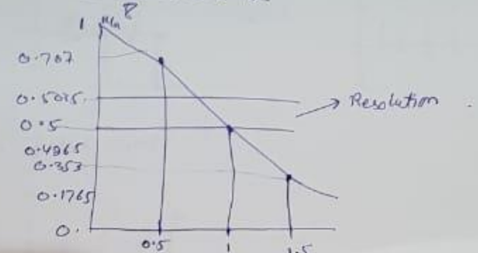
$L = 2^n$

$n = \text{bits} = 3$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



iii)

	Discrete time signal	Truncation	Reading	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4965	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

6.

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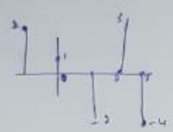
Q2

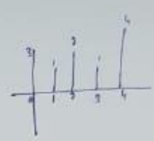
a) Determine the response of the ~~signal~~ system to the following input signal with given impulse response.

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

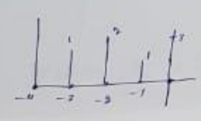
Sol: $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h(n-k)$



$h(k)$



$h(-k)$ folded signal



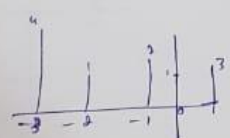
$$y[0] = \sum_{k=-1}^{0} x(-1)\,h(-1) + x(0)\,h(0)$$

$$y(0) = (2)(1) + (1)(3)$$

$$= 2+3 = 5$$

for $n = 1$

$h(1-k)$



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$y(1) = \sum_{k=-1}^{1} x(n)\ h(1-k)$

$= x(-1)h(-1) + x(0)h(0) + 0\ x(1)h(1).$

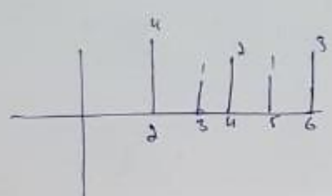
$+ (x) h(1) + x(2)x(2) + 0(x(3)h(3)$

$y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 + 3 - 12 = -4$

$n = 3$

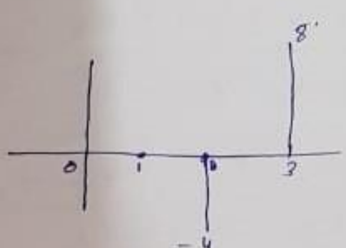
$h(3-k)$



$y(3) = \sum_{k=2}^{3} x(k)\ h(n-k)$

$= x(2)\ h(2) + x(3)h(3)$

$(3)(4) + (-4)(1) = 12 - 4 = 8.$



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Q3

i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right), & n < 0. \end{cases}$

<u>Sol:</u>

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

writing in the form of z-transform.

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n z^n - 1$$

using geometric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}} - 1$$

$$= \frac{1-\frac{1}{4}z^{-1} + 1-\frac{1}{1}^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} - 1$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1} - \left(1+\frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}\cancel{z^{-1}}\cancel{z}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

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$$= \frac{1 - \cancel{\frac{1}{3}z} + 1 - \cancel{\frac{1}{4}z^{-1}} - 1 + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}z^{-1}} + \frac{1}{12}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z)}$$

$$= \frac{1 + \frac{1}{12}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z)}$$

$$= \frac{\frac{13}{12}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z)}.$$

Hence The ROC is $\frac{1}{4} < |z| < 3$.

Q3 ii)

$$x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{else where.} \end{cases}$$

Soln

$$x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$$

In the form of z-transform.

$$X(z) = \sum_{n=0}^{\infty} (\tfrac{1}{2})^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

using geomatric series to simplyfie it.

$$= \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - 3z^{-1})}$$

$$= \frac{-\frac{5}{2}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - 3z^{-1})}$$

The ROC is $|z| > 3$.