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Section A

Paper Differential equation

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Semister 4th

Q NO. 1

Solve the following objective type questions.

i) The order of matrix A is $m \times p$ and the order of B is $p \times n$ Then the order of matrix AB is?

<u>Sol:</u> The order of matrix is equal to the no of its row multiply by no of column

So, $A = m \times p$ has "m" no of rows and p no of column

Similarly, $B = p \times n$

then its "p" no of Rows and n has no of column

Also the number of column in A is equal to the no of rows in B so there matrix are comformable for multiplication and there order will be

$$\boxed{AB = m \times n}$$

ii) The number of non zero rows in Echelon form?

One

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Sol For Singular Matrix $|B| = 0$

So, $|B| = 1 \times a - 4 \times 2 = 0$

$= a - 8 = 0$

So value of $a = 8$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= (2i)(-i) - (i)(i)$

$= -2i^2 - i^2$

we know that $i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$\boxed{|A| = 3}$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol If each element of a principal diagnal of a matrix is some non zero scalar and all other elements are zero then it is a scalar matrix so,

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix

vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{1}{y}dy = (1-2x)\,dx$$

Taking integral of Both sides.

$$\int \frac{1}{y}dy = \int (1-2x)\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\cancel{\ln y = 2x - 4x^2 + 2c}$$

$$\ln y = x - x^2 + C$$

vii) The order and degree of of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Sol The order of differential equation is the order of highest derivative known as differential co-efficient and Degree is the power of highest derivative so,

Order = 1

degree = 3

viii) The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

Order = 2

Degree = 1

(5)

ix) The differential equation

$2\frac{dy}{dx} + x^2 y = 2x+3$, $y(0) = 5$ is ?

Sol

$$2\frac{dy}{dx} + x^2 y = 2x+3$$

$$\int 2dy = \int (2x+3-x^2y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$y = \frac{\cancel{2}x^2}{\cancel{2}\times 2} + \frac{3x}{2} - \frac{y\,x^3}{3\times 2} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + c$$

put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

then

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$$

(6)

X) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Expand by R_1

$|A| = +1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a \begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2 \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$

$= 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c-b)$

$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

Q2 Part "A"

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor which are linear in a, b, c

Sol:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$
$+ c(a^2b^3 - a^3b^2)$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 -$
a^3b^2c

Common abc

$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$\Rightarrow abc[bc(c-b) - ac(c+a) + ab(b-a)]$

Ans

Q2 Part "B"

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:- $$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

charecteristic eq $\rightarrow |A - \lambda I| = 0 \rightarrow (A)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[(3-\lambda)(2-\lambda)-(-1)(-1)\right] + 1((-1)(2-\lambda)$$

$$-(-1)(-1)) - 1\left[(-1)(-1)-(-1)(3-\lambda)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \longrightarrow (a)$$

$$\Rightarrow +1\begin{vmatrix} -2 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$= \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow (b)$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$\Rightarrow -[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$= \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow (c)$

put a, b, c in eq (B)

$(2-\lambda)\left[-\lambda^3 + 8\lambda^2 - 18\lambda + 8\right] - \lambda^2 + 6\lambda - 8 - \lambda^2$
$+ 6\lambda - 8]$

$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda$
$-\lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2$
$-36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 16$

(4)

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By synthatic division

we get

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By facterization method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4)$

$\lambda = 4, \lambda = 4$

$\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4$

(1)

Q No. 3

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$x = 2, \quad y = 6$

Sol

$(x^2+3y^2)\,dx - 2xy\,dy = 0$

$(x^2+3y^2)\,dx = 2xy\,dy$

Divide both side by $2xy\,dx$

we get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (1)$$

Let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad — (a)$$

put a in (i)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

(2)

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$

we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take $\int$ on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$\Rightarrow \quad 1+v^2 = xc$

put $v = y/x$

$1+(y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3 c \longrightarrow$ (i)

put $x=2,\ y=6$ in eq (i)

$(4)+(36) = 8c$

$c = \frac{4}{8}$

$c = 5 \rightarrow$ put in eq (i)

so

$x^2+y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Taking "$\sqrt{\ }$" on both sides

$y = +x\sqrt{5x-1}, \ y = -x\sqrt{5x-1}$

or

$\boxed{y = \pm x\sqrt{5x-1}}$ Ans