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COURSE : Software (BS)
PAPER : Linear Algebra
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x ———— x ———— x ———— x ———— x

Attempt the following questions.

1- Solve the following systems of linear equation by Guass-Jordan Method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Solution :-

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_1 = \frac{1}{2}R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\xrightarrow[R_3 = R_3 - 3R_1]{R_2 = R_2 - R_1} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \xrightarrow{R_2 = \frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix}$$

$R_3 = R_3 + 2R_1$ $\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \xrightarrow[R_3 = -1/3 R_3]{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$.

$R_1 = R_1 - 2R_3$ $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow \left.\begin{bmatrix} x = 1 \\ y = 2 \\ z = 3 \end{bmatrix}\right\}$ Sol.

So

$z = 3$. Answer.

XX ——— X ——— X ——— X ——— XX

2- Reduce the matrix to normal form and find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Reduce matrix to redue row-echilon form.

Swap matrix Row $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

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Cancel leading co-efficient in row R_2 by performing.

$$R_2 \leftarrow R_2 - \frac{1}{3} R_1$$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in Row R_3 by performing.

$$R_3 \leftarrow R_3 - \frac{1}{3} R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number of all zero row.

$$\text{Rank of } \begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} = \mathbf{2}.$$

Answer.

XX ———— x ———— x ———— XX

3- Determine if the following homogeneous system has a non-trivial solution. Then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2\cancel{8}x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Solution:-

$$\begin{cases} 3x_1 + 5x_2 - 4x_3 = 0 \\ -3x_1 - 2\cancel{8}x_2 + 4x_3 = 0 \\ 6x_1 + x_2 - 8x_3 = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So, we have a sol of

$$x = \begin{bmatrix} 4/3\,s \\ 0 \\ s \end{bmatrix} = s\begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$ Answer.

XX————X————X————XX

4- Determine if the following system is consistent or not:

$$x_1 - (3^{rd} - ID)x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

ID = 15791
3rd = 7

Solution:-

$$x_1 - 7x_2 + x_3 = 0.$$
$$2x_2 - 7x_3 = 8.$$
$$5x_1 \qquad - 5x_3 = 10.$$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & +40 & -10 & 10 \end{array}\right] \sim R_3 - 5R_1.$$

$$\sim \left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right]; \quad \begin{array}{l} R_2/4 \\ \\ R_3/10 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & -15 \end{array}\right]; R_3 - 4R_2.$$

Consistent because of this triangle.

R/work.

$-15x^3 = -15$

$x_3 = 1.$

$x_1 = 7x^2 - x^3$

$x_1 = x(7x - 1x)$

$x_1 = 6x.$

$x_1 = 6.$

$x^2 - 4x^3 = 4.$

$x_2 = 4 + 4x^3.$

$x_2 = 8.$

$x_1 = 7x_2 + x_3 = 0$

$x_1 = 7x^2 - x^3.$

$x_1 = 6$ Answer.

xx ———— x ———— x ———— xx

5- Find The inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4^{th}\ Id \\ 5 & -2 & 7 \end{bmatrix}$$ by adjoint method.

Solution

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4^{th}\ ID \\ 5 & -2 & 7 \end{bmatrix}$$

ID: 15,791

4^{th} ID: 9.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 9 \\ 5 & -2 & 7 \end{bmatrix}$$

To find the inverse using the formula, we first determine the cofactors, C_{ij} of A.

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We have :

$C_{11} = \begin{vmatrix} -1 & 9 \\ -2 & 7 \end{vmatrix} = 11, \quad C_{12} = -\begin{vmatrix} 2 & 9 \\ 5 & 7 \end{vmatrix} = -31,$

$C_{13} = \begin{vmatrix} 2 & 9 \\ 5 & 7 \end{vmatrix} = 01.$

$C_{21} = -\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 38., \quad C_{22} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 21,$

$C_{23} = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -26.$

$C_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 9 \end{vmatrix} = 41, \quad C_{32} = -\begin{vmatrix} 3 & 5 \\ 2 & 9 \end{vmatrix} = 17.$

$C_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11$

Then the adjoint of A matrix A is

$$Adj(A) = A^t = \begin{bmatrix} 11 & 38 & 41 \\ -31 & -4 & 17 \\ 1 & -26 & 11 \end{bmatrix}$$

Using the formula, we obtain the inverse formula.

$$A^{-1} = \frac{1}{\det(A)} \quad \text{Adj}(A) = \begin{bmatrix} -11 & -38 & -41 \\ 31 & 4 & -17 \\ -1 & 26 & -11 \end{bmatrix}$$

Answer.

XX ———— X ———— X ———— XX