Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Assignment
Date:20/4/2020

Course Code:	MTH 102			**Course Title:**	Calculus and analytic geometry
Prerequisite:				**Instructor:**	HIMAYATULLAH
Module:	3	**Program:**	BEE	**Total Marks:**	30

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Q1.	(a)	. **Identify** $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y=\left(x+\frac{1}{x}\right)(x-\frac{1}{x}+1)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s=160t-16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y=x^4-2x^2+2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

Question No 1

Part (a)

Identify $\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Solution

$$\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Applying limit.

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0}$$

$$= \frac{\sqrt{2} - \sqrt{2}}{0}$$

$$\boxed{= \frac{0}{0}}$$

So we find.

$$\lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

xing and ÷ $\sqrt{2+h} + \sqrt{2}$

$$= \lim_{h\to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{(h)(\sqrt{2+h} + \sqrt{2})}$$ ∵ So square and √ cut each other.

$$= \lim_{h\to 0} \frac{2+h-2}{(h)(\sqrt{2+h}+\sqrt{2})}$$

$\because$ So 2 and -2 cut each other.

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{2+h}+\sqrt{2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

Apply Limit $_{h\to 0}$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$\boxed{= \frac{1}{2\sqrt{2}}}$$

Question 1:-
Part (b):-

Find the first order derivatives of the function

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right).$$

Sol

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

Apply derivatives.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

$$= (x+x^{-1})\frac{d}{dx}(x-x^{-1}+1) + (x-x^{-1}+1)\frac{d}{dx}(x+x^{-1})$$

$$= (x+x^{-1})(1+x^{-2}) + (x-x^{-1}+1)(1-x^{2})$$

$$= \left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right) + \left(x-\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$

$$= x + x\frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} + x - x\frac{1}{x^2} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2}$$

$$= \boxed{2x + 1 - \frac{1}{x^2} + \frac{1}{x^3}} \quad A$$

Question (2)

Part A:-

$$S = 160t - 16t^2 \text{ ft}$$

Solution

$$S = 160t - 16t^2 \text{ ft}$$

a)

Velocity is

$$v = \frac{ds}{dt}$$

$$\frac{d}{dt}(160t - 16t^2)$$

$$v = 160 - 32t$$

Maximum height.

$$V = 0$$

$$160 - 32t = 0$$

$$\frac{160}{32} = \frac{\cancel{32}t}{\cancel{32}t}$$

$$t = \frac{160}{32}$$

$$t = 5 \text{ sec.}$$

$$S_{max} = S(5) = 160(5) - 16(5)^2$$

$$\boxed{= 400 \text{ ft}}$$

(b)

Solution

$$S = 256 \text{ ft}$$

As we know

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$16(t^2 - 10t + 16) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 16$$

$\because$ 0 devide by something = 0

$$t^2 - 8t - 2t + 16 = 0$$

$t(t-8)-2(t-8)=0$

$(t-8)(t-2)=0$

$t-8=0 \qquad t-2=0$

$t=8 \qquad t=2$

$t_1 = 8\,sec \qquad t_2 = 2\,sec$

So

$v = 160 - 32t$

$t_1 = 8\,sec$

$V_{(8)} = 160 - 32(8)$

$= 160 - 256$

$\boxed{= -96\,m/s}$

$t_2 = 2\,sec$

$V_{(2)} = 160 - 32(2)$

$= 160 - 64$

$\boxed{= 96\,m/s}$.

(c)

So

$v = 160 - 32t$

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$

$a = 0 - 32\,m/s^2$

$\boxed{a = -32\,m/s^2}$

Question No(3):-

Part (a)

Does the curve $y = x^4 - 2x^2 + 2$ have nay horizental tangent if So where?

Solution:

$$y = x^4 - 2x^2 + 2$$

Apply derivatives on both sides

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx}x^4 - \frac{d}{dx}2x^2 + \frac{d}{dx}2$$

$$= 4x^2 - 4x + 0$$

$$\frac{dy}{dx} = 4x^2 - 4x$$

the tangent is horizental. $\frac{dy}{dx} = 0$

So

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad , \quad x^2 - 1 = 0$$

$$x = \frac{0}{4} \qquad x^2 = 1$$

$$\boxed{x = 0} \qquad \sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1}$$

So

$$x = 0 \quad , x = \pm 1$$

Correspondig point

$$y = x^4 - 2x^2 + 2$$

So $x=0$

$$y = x^4 - 2x^2 + 2$$
$$= (0)^4 - 2(0)^2 + 2$$
$$= 0 - 0 + 2$$
$$\boxed{y = 2}$$

$x = 1$

$$y = x^4 - 2x^2 + 2$$
$$= (1)^4 - 2(1)^2 + 2$$
$$= 1 - 2 + 2$$
$$\boxed{y = 1}$$

$x = -1$

$$y = x^4 - 2x^2 + 2$$
$$= (-1)^4 - 2(-1)^2 + 2$$
$$= 1 - 2 + 2$$
$$\boxed{= 1}$$

Hence.

$$\boxed{(0,2), (1,1), (-1,1)}$$