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Subject: Applied calculus

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Q=1

Sol:.

Given the ~~point~~ points.

$P(x_1, y_1, z_1) = P(4,1,3)$

and $(x_2, y_2, z_2) = Q(1,2,4)$

(i) Distance b/w P&Q

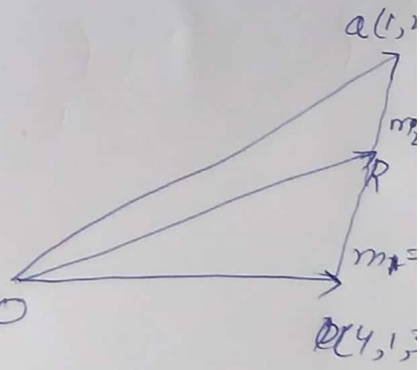
$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2+z_1)^2}$ By distance form

$|PQ| = \sqrt{(4-1)^2+(1-2)^2+(3-4)^2}$

$= \sqrt{3^2+(-1)^2+(-1)^2}$

$= \sqrt{9+1+1}$

$\Rightarrow |\overline{PQ}| = \sqrt{11}$



Let $\overrightarrow{OR}$ be the position vector of a point R which divides $\overrightarrow{PQ}$ in the ratio $m_1 : m_2 = 1:3$

Then $\dfrac{m_1 \overrightarrow{OP} + m_2 \overrightarrow{OQ}}{m_2+m_1}$

$$= \frac{1(i+2j+4k)+3(4i+j+3k)}{1+3}$$

$$= \frac{i+2j+4k+12i+3j+9k}{4}$$

$$\Rightarrow \overrightarrow{OR} = \frac{13i}{4} + \frac{5j}{4} + \frac{13k}{4}$$

or $xi+yj+zk = \frac{13}{4}i + \frac{5}{4}i + \frac{13}{4}k.$

Ans

Q=3

Sol: $\int_0^2 x^2 e^x \, dx$

$= x^2 \int e^x \, dx - \int \left\{ \frac{d}{dx}(x^2) \int e^x \, dx \right\} dx$

Integration by parts.

$= x^2 e^x - \int 2x \, e^x \, dx$

$= x^2 e^x - 2 \int x \, e^x \, dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left\{ \frac{d}{dx}(x) \int e^x dx \right\} \right.$

$= x^2 e^x - 2 \left(x e^x - \int e^x dx \right)$

$= x^2 e^x - 2x e^x + 2 \int e^x \, dx.$

$= x^2 e^x - 2x e^x + 2e^x$

$\Rightarrow \int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x$

There fore

$\int_0^2 x^2 e^x dx = \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= 2^2e^2 - 2(2)e^2 + 2e^2 - (0^2 \cdot e^0 - 2(0)e^0 + 2e^0)$

$= 4e^2 - 4e^2 + 2e^2 - (0 - 0 + 2 \times 1)$

$= 2e^2 - 2$

$= \int_0^2 x^2 e^2 \, dx = 2e^2 - 2$ Ans

Q=3 P B

Sol:

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx \rightarrow ①$$

Substitute : $\sqrt{x} = t$

Diff w.r.t x

$$\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = 2dt$$

limits

when $x = 1$ then $t = \sqrt{x} = \sqrt{1}$ $\quad t = 1$

when $x = 2$ then $t = \sqrt{x} = \sqrt{2}$ $\quad \boxed{t = \sqrt{2}}$

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$\therefore$ ① $\Rightarrow \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_1^{\sqrt{2}} \sin t \cdot 2\,dt$

$= 2\int_1^{\sqrt{2}} \sin t\,dt$

$= 2\left|-\cos t\right|_1^{\sqrt{2}}$

$= 2\left(-\cos\sqrt{2} + \cos(1)\right|$

$\Rightarrow \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\left\{-\cos\sqrt{2} + \cos(1)\right\}$ Ans

Q2:

Sol:- $\int \frac{4x^3 + 10x + 4}{2x^2 + x}\,dx \longrightarrow$ ①

$$
\begin{array}{r|l}
 & 2x - 1 \\
2x^2 + x & 4x^3 + 10x + 4 \\
 & \pm 4x^3 \quad \pm 2x^2 \\
\hline
 & -2x^2 + 10x + 4 \\
 & \mp 2x^2 \mp x \\
\hline
 & 11x + 4
\end{array}
$$

$$\therefore \frac{4x^3+10x+4}{2x^2+x} = 2x-1+\frac{11x+4}{2x^2+x}$$

$$\therefore \int \frac{4x^3+10x+4}{2x^2+x} = \int (2x-1)\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$I = I_1 + I_2 \longrightarrow (2)$$

Here

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x}+\frac{B}{2x+1} \longrightarrow (3)$$

or $11x+4 = A(2x+1)+Bx \longrightarrow (4)$

$(4) \Rightarrow 11(0)+4 = A\{2(0)+1\}+B(0)$

$$4 = A(1)+0$$

put $2x+1=0 \Rightarrow x = \frac{-1}{2}$ in 4

$(4) \Rightarrow 11\left(-\frac{1}{2}\right)+4 = A\left(x \times \frac{1}{8}+1\right)+B\left(\frac{-1}{2}\right)$

$$\frac{-11}{2}+4 = A(0) - \frac{B}{2}$$

$$\frac{-11+8}{2} = -\frac{B}{2}$$

$$\frac{-3}{\cancel{2}} = \frac{-B}{\cancel{2}} \Rightarrow B = 3$$

Now

$$\frac{11x+4}{2x^2+x} = \frac{4}{x} + \frac{3}{2x+1}$$

Now (2) $\Rightarrow$

$$\int \frac{4x^3+10x+4}{2x^2+4} = \int (2x-1)\,dx + \int \frac{11x+4}{2x^2+4}\,dx$$

$$= 2\int x\,dx - \int 1\,dx + \int \left(\frac{4}{x} + \frac{3}{2x+1} \right) dx$$

$$= \cancel{2}\left(\frac{x^2}{2}\right) - x + 4\ln(x) + \frac{3}{2}\ln(2x+1)$$

$$= x^2 - x + 4\ln(x) + \frac{3}{2}\ln(2x+1)$$

Ans.

Q=4

Sol:: Given

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \longrightarrow ①$$

The laplace equation for 3d is

$$\nabla^2 u = 0 \longrightarrow ②$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$② \Rightarrow \nabla^2 u = 0$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \longrightarrow ③$$

we have to prove eqtn ③ for the given function $u(x,y,z)$.

Now $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{-\frac{1}{2}}$$

$$= -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \, 2x$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[-x(x^2+y^2+z2)^{-3/2}\right]$$

$$= -x \cdot -\frac{3}{2}(x^2+y^2+z^2)^{-5/2} \cdot 2x$$

$$-(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \quad \text{—— (A)}$$

Now

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y} = \frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1/2}$$

$$= -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \cdot 2y$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

and

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left[-y(x^2+y^2+z^2)^{-3/2}\right]$$

$$= -y \cdot -\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{-5/2} \cdot \cancel{2}y$$

$$-(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$Q=4$

Now $\dfrac{\partial u}{\partial z} = \dfrac{\partial}{\partial z}(x^2+y^2+z^2)^{-1/2}$

$$= -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} \cdot 2x$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

and

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left[-z(x^2+y^2+z^2)^{-3/2}\right]$$

$$= -z \cdot -\frac{3}{2}(x^2+y^2+z^2)^{-5/2} \cdot 2z$$

$$-(x^2+y^2+z^2)^{-3/2}$$

$$= 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow$$

Now putting the value in (3)

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

$$3x(\cancel{x}^2+y^2+z^2)^{-5/2} \; (x^2+y^2+z^2)^{-3/2} + 3y(x^2+y^2+z^2)^{-5/2}$$

$$-(x^2+y^2+z^2)^{-3/2} + 3z(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow 3(x+y+z)(x^2+y^2+z^2)^{-5/2} - 3(x^2+y^2+z^2)^{-3/2} = 0$$

$$= -3(x^2+y^2+z^2)^{3/2}\left[1 - \frac{x^2}{x^2+y+z^2} \frac{-y^2}{x^2+y^2+z^2} - \frac{z^2}{x^2+y^2+z^2}\right]$$

$$= -3(x(x^2+y^2+z^2)^{-3/2}\left[\frac{x^2+y^2+z^2-x^2-y^2-z^2}{x^2+y^2+z^2}\right]$$

$$= -3(x^2+y^2+z^2)^{-3/2} \quad (0)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

Hence laplace are is statisfiend.