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SUBJECT: CALCULUS AND ANALYTICAL GEOMETY

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Q1:- (a) $\dfrac{2x^3 - 3x^2 + 5}{x^2 + 1}$

Sol:-

let $y = \dfrac{2x^3 - 3x^2 + 5}{x^2 + 1}$

differentiating with respect to x

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}(2x^3-3x^2+5) - (2x^3-3x^2+5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x^2-6x) - (2x^3-3x^2+5)2x}{(x^2+1)^2}$$

$$= \frac{6x^4 - 6x^3 + 6x^2 - 6x - (4x^4 - 6x^3 + 10x)}{(x^2+1)^2}$$

$$= \frac{6x^4 - \cancel{6x^3} + 6x^2 - 6x - 4x^4 + \cancel{6x^3} - 10x}{(x^2+1)^2}$$

$$= \frac{2x^4 + 6x^2 - 16x}{(x^2+1)^2}$$

$$= \frac{2x(x^3 + 3x - 8)}{(x^2+1)^2}$$

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(b):- $\frac{(x^2+1)^2}{x^2-1}$

Sol: let $y = \frac{(x^2+1)^2}{x^2-1}$ by Quotient Rule

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(2(x^2+1)2x) - (x^2+1)^2 2x}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)[2(x^2-1) - (x^2+1)]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(2x^2-2-x^2-1)}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

Q2:(a):- Find $\frac{dy}{dx}$ if $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$ using chain rule.

Solution:-

$$y = (1+2\sqrt{x})^3 \, x^{2/3}$$

Since it only has two variables so we will solve it with the help of product rule not by chain rule

$$\frac{dy}{dx} = 3(1+2\sqrt{x})^2 \left(0 + 2\tfrac{1}{2}x^{-1/2}\right) \cdot x^{2/3} + (1+2\sqrt{x})^3 \cdot \tfrac{2}{3} x^{1/3}$$

$$= 3(1+2\sqrt{x})^2 \cdot x^{2/3 - 1/2} + \frac{2}{3} \frac{(1+2\sqrt{x})^3}{x^{1/3}}$$

$$= 3(1+2\sqrt{x})^2 \cdot x^{1/6} + \frac{2}{3} \frac{(1+2\sqrt{x})^3}{x^{1/3}}$$

$$= (1+2\sqrt{x})^2 \left(3x^{1/6} + \frac{2}{3} \frac{(1+2\sqrt{x})}{\sqrt[3]{x}}\right)$$

Q2:-

(b)

$$y = \sqrt{\frac{1-x}{1+x}}$$

Sol:- outside function = $\frac{1}{2}$ or sqrt

inside function = $\frac{1-x}{1+x}$

Now by chain Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1-x}{1+x}\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\frac{1-x}{1+x}\right]^{\frac{1}{2}-1} \cdot \frac{d}{dx}\left[\frac{1-x}{1+x}\right]$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \left[\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1+x}{1-x}\right]^{\frac{1}{2}} \cdot \left[\frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}\right]$$

$$= \frac{1}{2}\left[\frac{1+x}{1-x}\right]^{\frac{1}{2}} \cdot \left[\frac{-2}{(1+x)^2}\right]$$

$$= -\left[\frac{1}{(1+x)^2}\right]\left[\frac{1+x}{1-x}\right]^{\frac{1}{2}}$$

$$= \frac{-1}{(1+x)^2}\sqrt{\frac{1+x}{1-x}}$$

or

$$\frac{-1}{(1+x)^2\sqrt{\frac{1-x}{1+x}}}$$

(Q 3) (a) $\int \frac{1}{\sqrt{x^3}} dx$

$$\Rightarrow = \int \frac{1}{x^{3/2}} dx \quad \left(\because \sqrt{x^3} = x^{3/2} \right)$$

$$= \int x^{-3/2} dx$$

Applying formula $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$$= \frac{x^{-3/2+1}}{-3/2+1} + c$$

$$= \frac{x^{\frac{-3+2}{2}}}{\frac{-3+2}{2}} + c$$

$$= \frac{x^{-1/2}}{-1/2} + c$$

$$= \frac{1}{-\frac{1}{2}\sqrt{x}} + c$$

$$= -\frac{2}{\sqrt{x}} + c$$

Q3 (b) $\int \frac{1}{(6x+7)^6} dx$ —— (1)

Sol: by substitution method.

let $t = 6x+7$ —— (A)

$\frac{dt}{dx} = 6$

$dt = 6\,dx$ —— (B)

multiplying and dividing 6 ~~to~~ (1)

$\Rightarrow = \int \frac{6}{6} \cdot \frac{1}{(6x+7)^6} dx$

$= \frac{1}{6} \int \frac{6\,dx}{(6x+7)^6}$

putting (A) and (B) we get

$= \frac{1}{6} \int \frac{dt}{t^6}$

$= \frac{1}{6} \int t^{-6}\, dt$

$= \frac{1}{6} \cdot \frac{t^{-6+1}}{-6+1}$

$= \frac{1}{6} \cdot \frac{t^{-5}}{-5} + c$

$= \frac{1}{-30} \cdot (6x+7)^{-5} + c$

$= -\frac{1}{30(6x+7)^5} + c$