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Qno 1. find the root of the equation $x^3 + 3.993 \times 10^{-4} = 0.165x^2$ use newton raphson method with $x_0 = 0.02$

Sol:

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Arrange the equation first

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

$$\rightarrow \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

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$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$x_2 = 0.058$$

$$\rightarrow \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$$x_3 = 0.061$$

Q2: Use the numbers $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ find the lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ at $x = 3$

Sol:

formula :

$$y = \frac{(x-x_1)(x-x_2) \cdots (x-x_n)}{(x_0-x_1)(x_0-x_2) \cdots (x_0-x_n)} y_0$$

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Given:

$x_0 = 2$

$x_1 = 2.75$

$x_2 = 4$

So,

$f(x) = \frac{1}{x}$

$y_0 = 0.5$

$y_1 = 0.36$

$y_2 = 0.25$

$x_0 = 2$	$y_0 = 0.5$
$x_1 = 2.75$	$y_1 = 0.36$
$x_2 = 4$	$y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x-x_1)} y_2$$

Put in the formula :-

(P.T.O)

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$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)}(0.36) + \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$y = (-0.833) + 0.384 + 0.025$$

$$y = -0.424 \quad \text{ANSWER.}$$

Q.No : 3

Complete the divided difference table for the given data & construct the interpolating polynomial that uses all this data.

x	y
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

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table

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_i)$	$f(x_{i-4} \dots x_i)$
1.0	0.7651977				
		-0.4837056			
1.3	0.6200860		-0.108734		
		-0.548946		0.0658785	
1.6	0.4554022		-0.049443		0.0018 2375.
		-0.578612		0.068067	
1.9	0.2818186		0.011818		
		-0.571521			
2.2	0.1103623				

i $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$f(x_0, x_1) = -0.4837056$

2) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$f(x_1, x_2) = -0.548946$

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3) $f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$f(x_2, x_3) = -0.578612$

4) $f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$f(x_3, x_4) = 0.571521$

3RD divided difference.

i $f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$

$= \frac{0.0499433 - (-0.108734)}{1.9 - 1}$

F

Second divided difference.

$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$= -0.049443$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.011818$$

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.049443 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

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$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.011818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.068067.$$

fourth divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.068067 - 0.06587785}{2.2 - 1}$$

$$= 0.00182375$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x - 1) - 0.4837056 + (x - 1)(x - 1.3)(-0.108734) + (x - 1)(x - 1.3)(x - 1.6)(0.06587785) + (x - 1)(x - 1.3)(x - 1.6)(x - 1.9)(0.00182375)$$