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Summer Paper final

Linear Algebra

Q No: 7
(A part)

The non parallel vectors:

$P_1P_2 = (-3, 2, 3)$
$P_1P_2 \quad (3, -1, 3)$

$(-1, 0, 3) \; (2, -2, 1)$
$(-1, 0, 3) - 2, +2, -1$
$(-3, +2, 2)$

The perpendicular vector is:-
$m = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_2}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$\overrightarrow{P_1P_2} \sqrt{(x_2-x_1)+(y_2-y_1)^2}$
$\overrightarrow{P_1P_2} = \sqrt{(-1-2)}$

$m = i(6+2) - j(-9-6) + k(3-6)$
$n = 8i + 15j - 3k$
$m = (8, 15, -3)$

Now $P_1(x_0, y_0, z_0) = (2, -3, 1)$
$m(a, b, c) = 8, 15, -3)$

So equation of plane is
$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$8(x-2) + 15(y+2) - 3(z-1) = 0$
$8x + 15y - 3z - 16 + 30 + 3 = 0$
$8x + 15y - 3z + 17 = 0$

Q1 (b) $x = 2-3t$

$y = 3+t$

$z = 2-4t$

Solution

$x-2 = -3t \rightarrow t = \frac{x-2}{-3}$

$y = 3+t \Rightarrow t = \frac{y-3}{1}$

$z-2 = -4t \Rightarrow t = \frac{z-2}{4}$

So $\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{4}$

For 1st plane takes 1st and 2nd

$\frac{x-2}{-3} = \frac{y-3}{1}$

$x-2 = -3y+9$

$x+3y-11=0$

Q2.

$L(x, y) = ((x+1), y, x+y)$ illustrate that L is linear transformation.

Sol

$L(x, y) = (x+1, y, x+y)$

let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$U+V = (x_1, y_1) + (x_2 y_2)$

$U+V = (x_1 + x_2, y_1 + y_2)$

$L(U+V) \quad L(x_1 + x_2, y_1 + y_2)$

$L(U+V) = (x_1 + x_2 + 1, y, x_1 + x_2 + y_1 + y_2)$

given that $u = (x+y)$

$L(u) = L(x_1, y_1) = (x_2 + 1, y_i, x_2 + y_2)$

$L(u) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$

$L(u) = L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

Since $1 \neq 2$

Q3 $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

So,

$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

$X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 + 54 - 32 \\ 154 - 108 - 54 \\ -77 + 54 + 38 \end{bmatrix}$

$$X_1 = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 23 \end{bmatrix} = \begin{bmatrix} 28 \\ 1 \\ 16 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

$$X_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

PHOTOGRAPANSANS

Question No.12

$(-1, 3, 2)$ $n =$ $(0, 1, -3)$

Solution

Eq. of plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

Given that

$P = (x_0, y_0, z_0) = (-1, 3, 2)$

$n = (a, b, c) = (0, 1, -3)$

So, $0(x-(-1)) + 1(y-3) - 3(z-2)$

$0(x+1) + 1(y-3) - 3(z-2)$

$\downarrow$

0

$+ y - 3 - 3z + 6$

$\Rightarrow y - 3z - 3 + 0$

$\Rightarrow \boxed{y - 3z + 3}$ Ans

Scanned with CamScanner

Question # 5

Solution

Since we know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 \rightarrow ①$$

$$-2x_1 + 4x_2 = \lambda x_1 \longrightarrow ①$$

So, $x_1 - \lambda x_1 + x_2 = 0$

$= (1-\lambda)\, x_1 + x_2 = 0$

& $-2x1 + 4x_2 - \lambda x = 0$

$= -2x_1 + (4-\lambda)\, x_2 = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$= -2x1 + 4x_2 = 2x_2 \rightarrow ⑪$

$= -2x_1 + 2x_2 = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = \gamma$ then $x_2 = \gamma$

So $\quad x_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix}$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2 \rightarrow ⑪$

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = 1/2\, x^2$

let $x^2 = \gamma$

where $\gamma \neq 0$