Course Title: Electrical Network Analysis

Module: 4th

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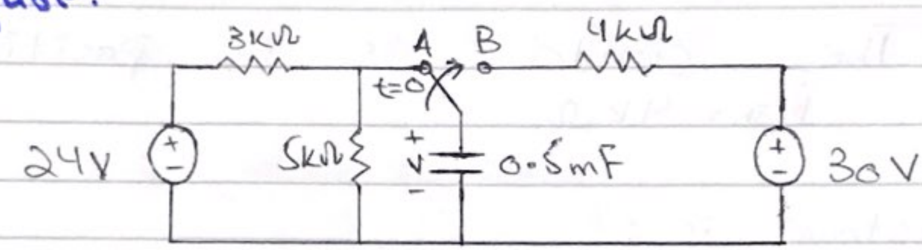
Question No: 1

The switch in figure has been in position A for a long time. At $t=0$ the switch moves to B. Determine $v(t)$ for $t>0$ & calculate it's value at $t=$ 2s & 8s.

Figure:



Solution:

For $t<0$:

The switch is at position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across 5kΩ resistor. Hence the across the capacitor just before $t=0$ is obtained by voltage division as;

$$v(0^-) = \frac{5}{5+3}(24) = 15\text{ V}$$

As the capacitor can't change instantaneously.

$$V(0) = V(0^+) = 15\text{ V}.$$

For $t > 0$:

The switch is in position B. The $R_{in} = 4k\Omega$

Time constant is ;

$$\tau = R_{in}C = 4 \times 10^3 \times 0.5 \times 10^{-3}$$
$$\tau = 2s$$

Since the capacitor acts like an open circuit to dc at steady state;

$$V(\infty) = 30\text{ V}.$$

Thus

$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

$$= 30 + (15 - 30)e^{-t/2}.$$
$$= (30 - 15e^{-0.5t})\text{ V}$$

At $t = 2$:

$$V(2) = 30 - 15e^{-\frac{2}{2}}$$
$$= 30 - 15e^{-1}$$
$$V(2) = 24.48\ V$$

At $t = 8$:

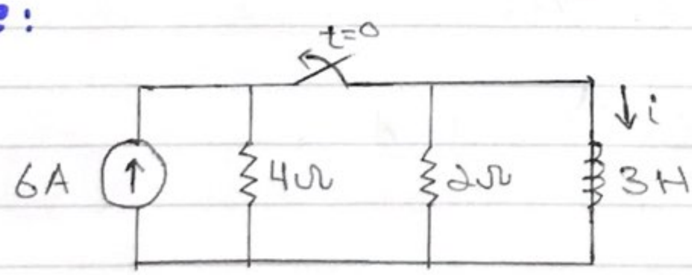
$$V(8) = 30 - 15e^{-\frac{8}{2}}$$
$$= 30 - 15e^{-4}$$

$$\cancel{V(8) = 29.7}$$
$$V(8) = 29.72$$

Question No: 2:

Determine the inductor current for both $t>0$ & $t<0$ for the circuit.

Figure:



Solution:

For $t < 0$:

The switch is closed & inductor acts as short circuit

Therefore inductor current

$$i = 6A$$

For $t > 0$:

The switch is opened & time constant $\tau = \frac{L}{R}$

$$\tau = \frac{3}{2}$$

Now the inductor current $i(t) = e^{-\frac{t}{\tau}}$

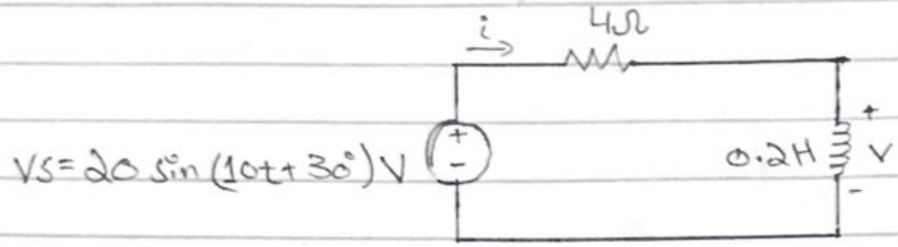
$$i(t) = 6e^{-\frac{t}{3/2}}$$

$$i(t) = 6e^{-2t/3}\, u(t)\ A.$$

Question No 5:

Find $v(t)$ & $i(t)$ in the ~~current~~ circuit in figure.

Figure:



Solution:

For $i(t)$

from the voltage source

$$V_s = 20\sin(10t + 30^\circ)\,V$$

$$V_s = 20\cos(10t + 30^\circ - 90^\circ)\,V$$

$$V_s = 20\cos(10t - 60^\circ)\,V$$

$$V_s = 20\angle -60^\circ\,V$$

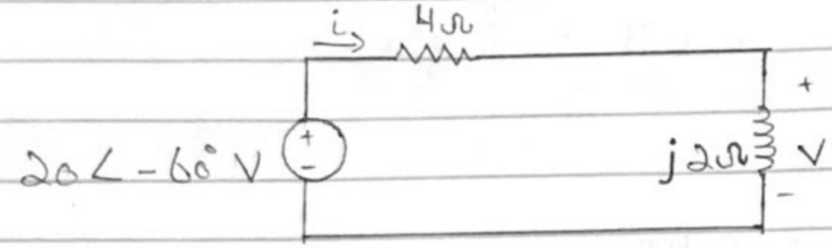
$$\omega = 10\ \text{rad/sec}$$

$$X_L = j\omega L$$

$$0.2H = j \times 10 \times 0.2$$

$$0.2H = j2\Omega$$

Given circuit can be represented As;



from the above circuit,

$$Z = 4 + j2\Omega$$

Hence, the current is,

$$i = \frac{20\angle -60^\circ}{4+j2}$$

$$i = \frac{20\angle -60^\circ}{\sqrt{4^2+2^2}\angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$i = \frac{20\angle -60^\circ}{4.472\angle 26.57^\circ}$$

$$i = 4.472\angle -86.57^\circ$$

Converting this into time domain.

$i(t) = 4.472 \cos(10t - 86.57^\circ)$

$i(t) = 4.472 \sin(10t - 86.57^\circ + 90^\circ)$

$i(t) = 4.472 \sin(10t + 3.43^\circ) A$

For $v(t)$:

From the circuit
Voltage across the inductor is,

$V = j2 \times i$

$V = j2 \times (4.472 \angle 86.57^\circ)$

Converting polar form to rectangular form we get;

$V = j2 \times (0.26756 - j4464)$
$V = 8.928 + j0.53512$

Converting rectangular form to polar form

$$V = \left(\sqrt{(8.926)^2 + (0.53512)^2}\right) \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$$

$$V = 8.944 \angle 3.4 \, V$$

Converting this into time domain

$$v(t) = 8.944 \cos(10t + 3.4^\circ)$$

$$v(t) = 8.944 \sin(10t + 3.4^\circ + 90^\circ)$$

$$v(t) = 8.944 \sin(10t + 93.4^\circ) \, V$$

Question No 3:

A series RLC circuit is described by

$$L\frac{d^2t}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5H$, $R = 4\Omega$ & $C = 0.2F$.

Let $i(0) = 1A$, $\frac{di(0)}{dt} = 0$

Solution:

The step response of the branch voltage of the given RLC circuit is described by;

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divide by L;

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{10}{L}$$

xing by $\frac{C}{C}$

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{10C}{LC}$$

As $C = 0.2F$, thus;

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute;

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \dots (i)$$

The general form for source-free RLC is given by.

$$\frac{d^2i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{1}{LC}\,i = \frac{I_s}{LC} \quad \cdots (2)$$

Comparing 1 & 2 we get,

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

From (3), α is given by;

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/sec} \rightarrow (6)$$

The natural frequency ω_0 is given by;

$$\omega_0 = \sqrt{\frac{1}{LC}} \rightarrow$$

From (4),

$$\omega_0 = \sqrt{10} \text{ rad/sec} \rightarrow (7)$$

From (6) & (7):

$$\because \alpha > \omega_0$$

$\therefore$ The circuit is overdamped

The roots of characteristic equation are given by ;

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 + \sqrt{6} \text{ rad/s}$$

And,

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$= -4 - \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 - \sqrt{6} \text{ rad/s}$$

From (5), the steady current is given by

$$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2\,A \rightarrow (8)$$

The current for overdamped is given by;

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \rightarrow (9)$$

Substitute $t=0$:

$$i(0) = I_s + A_1 + A_2$$

Substitute:

$$1 = 2 + A_1 + A_2$$

Thus,

$$A_1 + A_2 = -1 \rightarrow (10)$$

from 9, find $\frac{di(t)}{dt}$,

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Substitute $t=0$;

$$\cancel{\frac{di(0)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}}$$

$$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$$

Substitute the values;

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \rightarrow (11)$$

Solving (10) & (11) simultaneously

$$A_1 = -1.316$$
$$A_2 = 0.316$$

Substitute in (9)

$$i(t) = 2 - 1.316\,e^{(-4+\sqrt{6})t} + 0.316\,e^{(-4-\sqrt{6})t}\ A$$

Question No 4:

A series RLC circuit has

$$R = 100\,\Omega$$
$$L = 240\,H$$
$$C = 10\,mF$$

if the input voltage is $v(t) = 10\cos 2t$, find the current flowing through the circuit.

Solution:

The input voltage is,

$$v(t) = 10 \cos 2t \ V$$

Amplitude $= V_m = 10\,V$

Angular frequency $= 2$ rad/s

Phase angle, $\phi = 0^\circ$

So

Phasor for the voltage $v(t)$:

$v(t) = 10 \angle 0^\circ \ V$

Now for inductive reactance.

$X_L = \omega L$

So $\omega = 2$ rad/s, $L = 240\,H$

$X_L = (2)(240)$

$X_L = 480\,\Omega$

Now for capacitive Reactance

$$X_C = \frac{1}{\omega C}$$

$\omega = 2$ rad/s , $C = 10$ mf

$$\frac{1}{2(10 \times 10^{-3})}$$

$$\frac{1}{2 \times 10^{-2}}$$

$$\frac{1 \times 10^{2}}{2}$$

$$X_C = 50\ \Omega$$

Now for impedence;

$$Z = R + jX_L - jX_C$$

$R = 100\ \Omega$, $X_L = 480\ \Omega$, $X_C = 50\ \Omega$

puting in equation

$Z = (100 + 480 - 50)\,\Omega$

$Z = (100 + j430)\,\Omega$

Represent "Z" in Phasor form

$$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$$

$$= \sqrt{10{,}000 + 184{,}900} \angle \tan^{-1}(4.3)$$

$$= \sqrt{194{,}900} \angle \tan^{-1}(4.3)$$

$$Z = 441.47 \angle 76.90^\circ \,\Omega$$

Now for current flowing in the circuit;

$$i = \frac{v(t)}{Z}$$

$v(t) = 10 \angle 0^\circ$, $Z = 441.47 \angle 76.90^\circ \,\Omega$

putting in equation

$$i = \frac{10 \angle 0^\circ \text{ V}}{441.47 \angle 76.9^\circ \ \Omega}$$

$$i = \frac{10}{441.47} \angle [0 - 76.9^\circ] \text{ A}$$

$$= 22.6 \times 10^{-3} \angle -76.9^\circ \text{ A}$$

$$= 22.6 \angle -76.9^\circ \text{ mA}$$

So,

The general expression for "i"

$$i = 22.6 \cos(2t - 76.9^\circ) \text{ mA}.$$