

NAME

SOHAEL AHMED

ID

7907.

SUBJECT

D. EQUATION.

SEMESTER

4TH

SUBMITTED  
TO

MADAM SALOMATA.

MAZHER.

Date

13-06-2020.

QUESTION 01

The Cauchy Euler equation.

①  $x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$

Solution ::

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 2)y = 10x + 10x^{-1} \rightarrow \textcircled{1}$$

Let  $x = e^t \Rightarrow t = \ln x$

$$xD = D$$

~~$x^2 D^2 = D(D-1)$~~

$$x^2 D^2 = D(D-1) = D^2 - D$$

$$x^3 D^3 = D(D-1)(D-2)$$

Substituting into eq ①.

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10x + 10x^{-1}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

Using synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$0^2 - 20 + 2 = 0$$

Now using Quadratic Formula.

$$a = 1, b = -2, c = 2$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2(1)}$$

$$\Delta = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-1 \times 4}}{2}$$

$$\Delta = \frac{2 \pm 2i}{2}$$

$$\Delta = 1 \pm i$$

$$\Delta = 1 \pm i$$

Since roots are complex.

$$y_c = e^{-t} (c_1 \cos t + c_2 \sin t).$$

→ Now Particular Integration.

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot \frac{10}{e^t}$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (-1)^2 + 2}$$

$$\frac{\int 10e^t}{2} + \frac{\int 10e^{-t}}{2}$$

$$y_p = 5e^t + 5e^{-t}.$$

⇒ General Solution:

$$y = y_c + y_p$$

$$y = e^{-t} (c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}.$$

$$\text{put } e^t = x \text{ \& } t = \ln x$$

$$y = e^{-x} (c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$$

Ans.

# QUESTION 2:

9

$$x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} - 15y = x^4$$

SOLUTION :-

$$\text{Let } \frac{d}{dx} = D$$

$$x^3 D^3 y + 4x^2 D^2 y - 5x D y - 15y = x^4$$

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15)y = x^4$$

$$\text{let } x = et \Rightarrow t = \ln x$$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

$$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$$

Now substituting.

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15)y = x^4$$

$$D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$$

$$D^3 + D^2 - 7D - 15 = e^{4t}$$

Synthetic Division.

5	1	+1	-7	-15
		3	12	15
	1	4	5	0

$$\Delta^2 + 4\Delta + 5 = 0.$$

Quadratic formula.

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$= 2 \left( \frac{-2 \pm i}{2} \right)$$

$$\Delta = -2 \pm i$$

$$y_c = e^{3x} (C_1 \cos t + C_2 \sin t)$$

For  $y_p = ?$

$$y_p = \frac{1}{D^3 + 4D^2 - 7D - 15}, e^{3t}.$$

$$= \frac{1}{(4)^3 + (4^2) - 7(4) - 15} e^{4t}.$$

$$= \frac{1}{80-43} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}$$

Hence

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

again put  $t = \ln x$  &  $x = \ln x$

$$y = e^{2x} (c_1 \cos(\ln x) + c_2 \sin(\ln x)) + \frac{1}{37} e^{4x}$$

Ans.

QUESTION 3:  $x^2 y'' + 2xy' - by = 10x^2$

Solution -  $y(1) = 1$  &  $y'(1) = -6$

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - by = 10x^2$$

$$\Rightarrow x^2 d^2/dx^2 + 2x d/dx - b)y = 10x^2$$

put  $xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta-1) \Delta^2 = \Delta$

meet &  $\log x = t$

$$\Delta^2 - \Delta + 2\Delta - b)y = 10e^{2t} \Rightarrow \text{The Quadratic eq}$$
  
$$x^2 + x - b = 0 \Rightarrow \Delta^2 + 3\Delta - 2b - 6 = 0$$

(9)

$$\Rightarrow \Delta(\Delta+3) - 2(\Delta+3) = 0$$

$$\Rightarrow (\Delta+3)(\Delta-2) = 0$$

$$\Delta + 3 = 0, \Delta - 2 = 0$$

$$\Delta = 2, \Delta = -3$$

Since roots are real and distinct

For  $y_c = ?$

$$y_c = C_1 e^{-3t} + C_2 e^{2t}$$

For  $y_p = ?$

$$y_p = \frac{1}{\Delta^2 - \Delta - 6} \cdot 10 e^{2t}$$

$$= \frac{10}{\Delta^2 - \Delta - 6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \text{ fails}$$

Now

$$10 \frac{1}{d/d\Delta(\Delta^2 + \Delta - 6)} e^{2t}$$



$$\Rightarrow 10 \frac{t}{2\Delta+1} e^{2t} \quad (10)$$

$$= 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$y_p = 2te^{2t}$$

General solution

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2 \quad (B)$$

Put  $y(1) = 1$  i.e.  $x=1, y=1$  in (B)

$$1 = c_1 (1)^{-3} + c_2 (1)^2 + 2 \log(1)$$

$$1 = c_1 + c_2 \quad \longrightarrow \quad (C)$$

Now differentiate eq (B) w.r.t  $x$ .

(11)

$$y' = -3c_1 x^{-4} + 2(2x + \frac{2}{x}) (x^2) + 4x \log x$$

Now put  $y'(c_1) = -6$  i.e.  $y' = -6$  and  $x = -6$ .

$$-6 = -3c_1 + 2(2 + 2) + 0$$

$$\Rightarrow -6 = -3c_1 + 2(2 + 2)$$

$$\Rightarrow -6 - 2 = -3c_1 + 2(2 + 2)$$

$$-8 = -3c_1 + 2c_2 \quad \text{--- (1)}$$

Multiplying eq (1) with (2) and adding from (2)

$$\begin{array}{r} 2c_1 + 2c_2 = 2 \\ + 3c_1 - 2c_2 = -8 \\ \hline 5c_1 = 10 \end{array}$$

$$c_1 = \frac{10}{5} \quad \boxed{c_1 = 2}$$

$$-8 = -3(2) + 2c_2$$

$$-8 = -6 + 2c_2$$

$$2c_2 = -8 + 6$$

$$2c_2 = -2$$

(12)

$$C_2 = \frac{-2}{2} \cdot 1$$

$$\boxed{C_2 = -1}$$

now put the value of  $C_1$  and  $C_2$  in eq (B).

$$y = 2x^{-3} - x^2 + 2 \ln x \cdot x(x^2)$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x \text{ ANS}$$

---

$$y_p = \frac{1}{s^2 + 6s + 5} e^{st} \quad (15)$$

$$= \frac{1}{(s)^2 + 6(s) + 5} e^{st}$$

$$= \frac{1}{60} e^{st}$$

Now General Solution is

$$y = y_c + y_p$$

$$y = C_1 e^{-st} + C_2 e^{-t} + \frac{1}{60} e^{st}$$

$$y = C_1 x^{-s} + C_2 x^{-1} + \frac{1}{60} x^s \rightarrow (B)$$

$x=0$  put in this equation

$$x=0 \text{ in eq (B)} \quad e^0 = 1$$

put  $y(0) = 2$  i.e.  $y = 2$  and  $x = 2$

$$2 = C_1 (2)^{-s} + C_2 (2)^{-1} + \frac{1}{60} (2)^s$$

(16)

$$2 = -32c_1 - 2c_2 + \frac{1}{60} \left( \frac{2}{15} \right)$$

$$2 = -32c_1 - 2c_2 + \frac{2}{15}$$

$$2 - \frac{2}{15} = -32c_1 - 2c_2$$

$$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow \textcircled{C}$$

Now differentiate eq(B) w.r.t(x)

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4 \rightarrow$$

Put  $y'(1) = 2$  i.e.  $y' = 2$  and  $x = 2$  in above equation

$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12} (2)^4$$

$$2 = -5c_1 (-64) - c_2 (4) + \frac{1}{12} (16)$$

$$2 = 320c_1 + 4c_2 + \frac{4}{3}$$

(17)

$$\Rightarrow 2 - \frac{4}{3} = 320c_1 + 4c_2.$$

$$\Rightarrow \frac{2}{3} = 320c_1 + 4c_2 \rightarrow \textcircled{D}$$

Ming eq (C) with 2 and then -ing eq (C)

from (D)

$$\frac{-44}{15} = 64c_1 + 4c_2.$$

$$\frac{-44}{15} = 64c_1 + 4c_1$$

$$\begin{array}{r} + \frac{2}{3} = +320c_1 + 4c_2 \\ + \hline \end{array}$$

$$\frac{34}{15} = -256c_1.$$

$$c_1 = \frac{34 \times 256}{15}$$

$$\boxed{c_1 = 580}$$

Put the value of  $c_1$  in eq (C).

$$\frac{22}{15} = -32(580) - 2c_2.$$

(13)

$$\Rightarrow \frac{22}{15} = -18560 - 2C_2.$$

$$\Rightarrow \frac{22}{15} + 18560 = -2C_2$$

$$\Rightarrow \frac{18561}{-2} = C_2.$$

$$\boxed{-9280 = C_2}$$

Now put the value of  $C_1$  and  $C_2$  in eq (13).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$$

Ans

QUESTION #03 (11)

$$(x+1)^2 y'' - 3(x+1)'y' + 4y = x^2$$

Solution:

$$(x+1)^2 \frac{d^2 y}{dx^2} - 3(x+1) \frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left( (x+1)^2 \frac{d^2}{dx^2} - 3(x+1) \frac{d}{dx} + 4 \right) y = x^2$$

$$\Rightarrow \left[ (x+1)^2 D^2 - 3(x+1)D + 4 \right] y = x^2 \rightarrow (A)$$

$$\text{Put } (x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$$

$$x = e^t \text{ in eq (A)}$$

$$\Rightarrow [\Delta^2 - \Delta - 3\Delta + 4] y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4) y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4) z = e^{2t}$$

for yc we find the roots

$$D^2 - 4D + 4 = 0$$

$$D^2 - 2D - 2D + 4 = 0$$

$$D(D-2) - 2(D-2) = 0$$



(20)

$$\Delta - 2 = 0, \Delta = 2.$$

$$\Delta - 2 = 0, \Delta = 2$$

so the roots are real and repeated.

The general solution are.

$$y = (C_1 + C_2 x)^{m_1}$$

$$y = (C_1 + C_3 x)^{2m_1}$$

For  $y_p = ?$

$$y_p = \frac{1}{\Delta^2 - 4\Delta + 4}$$

$$\begin{aligned} (2)^2 - 4(2) + 4 \\ \Rightarrow 0 \end{aligned}$$

$$y_p = \frac{2}{2\Delta - 4} e^{2t}$$

if we put 2

$$2\Delta - 4 \Rightarrow 2(2) - 4 = 0$$

we take again derivative.

$$y_p = \frac{2}{2} \cdot e^{2t}$$

$$y = (C_1 + C_2 x)^{2t} + e^{2t}$$

Ans

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm \sqrt{4^2}}{2}$$

$$= \frac{-3 \pm 2}{2}$$

$D = -3 \pm 2$  Since roots are real and distinct

$$y_c = C_1 e^{-3t} + C_2 e^{-t}$$

For  $y_p = ?$

(13)

QUESTION # 01

$$x^2 y'' + 7xy' + 5y = x^5$$

$$y(0) = 2 \text{ and } y'(1) = 2$$

SOLUTION :-

$$x^2 \frac{dy^2}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left( x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5 \right) y = x^5 \quad \text{--- (A)}$$

$$\text{Put } xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$$

$$x = et \Rightarrow \log x = t \text{ in eq (A)}$$

$$\Rightarrow (\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$$

$$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}$$

By Quadratic Formula,

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$