(1)

Name: Hazrat Bilal ID: 14622

Course Title: Electromagnetic field.

Department: Electrical Engineering.

Q No1:-

(e) $G = 2a_x - 3a_y + 4a_z$.

Solution:

(a) in the direction of $a_\rho$ = the incremental work is given by $dW = -qE \cdot dL$, where in this case $dL = d\rho a_\rho = 6\times10^{-6} a_\rho$. Thus

$$dW = -(20\times10^{-6} C)(100 V/m)(6\times10^{-6} m)$$

$$= -12\times10^{-9} J$$

$$\boxed{= -12 nJ.}$$

b) in the direction of $a_\Phi$ = In this case $dL = 2d\Phi a_\Phi = 6\times10^{-6}$ and so,

$$dW = -(20\times10^{-6})(-200)(6\times10^{-6})$$

② 

$= 2.4 \times 10^{-8}\ J$

$= \boxed{24\ nJ}$

c) in the direction of [illegible]

$dL = dz\,a_z = 6 \times 10^{-6}\,a_z$

$dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6})$

$= -3.6 \times 10^{-8}\ J$

$= \boxed{-36\ nJ}$

d) in the direction of $E$.

$$a_E = \frac{100a_x - 200a_y + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$

$= 0.267a_x - 0.534a_y + 0.802a_z$

thus

$dW = -(20 \times 10^{-6})\{100a_x - 200a_y + 300a_z\} \cdot \{0.267a_x - 0.534a_y$
$+ 0.802a_z\} \times 6 \times 10^{-6}$

$= \boxed{-44.9\ nJ}$

In the direction of $G = 2a_x - 3a_y + 4a_z$.

$$a_a = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}}$$

$$= 0.371a_x - 0.557a_y + 0.743a_z.$$

so Now.

$$dW = -(20 \times 10^{-6})[100a_\rho - 200a_\phi + 300a_z] \cdot [0.371a_x - 0.557a_y + 0.743a_z](6 \times 10^{-6}).$$

$$= -(20 \times 10^{-6})[37.1(a_\rho \cdot a_x) - 55(a_\rho \cdot a_y) - 74.2(a_\phi \cdot a_x) + 111.4(a_\phi \cdot a_y) + 222.9](6 \times 10^{-6})$$

where at P, $(a_\rho \cdot a_x) = (a_\phi \cdot a_y) = \cos(40°) = 0.76$

$$(a_\rho \cdot a_y) = \sin(40°) = 0.643$$

$$(a_\phi \cdot a_x) = -\sin(40°) = -0.643$$

Now substituting these result in.

$$dW = -(20 \times 10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6 \times 10^{-6})$$

$$= \boxed{-41.8\,nJ}$$

④

Q No 2 :- Let $E = 10[\sin \pi/6)a_x + 5\sin \pi/6\, a_y + 10\cos \pi/6\, a_z\, ]$.

Solution :-

a) $E_p = -10\{\sin(\pi/6)a_x + 5\sin(\pi/6)a_y + 10\cos(\pi/6)a_z\}$

$= -[5a_x + 25a_y + 50\sqrt{3}\, a_z]$.

b) $dw_x = -qE \cdot dLa_x$

$= -2\times10^{-9}(-5)(10^{-3}) = 10^{-11}J$

$= \boxed{10\,PJ}$

c) of $a_y$?

$dw_y = -qE \cdot dLa_y$

$= -2\times10^{-9}(-25)(-10^{-3}) = 50^{-11}J$

$= \boxed{50\,PJ}$

d) of $a_z$.

$dw_z = -qE \cdot dLa_z$.

$= -2\times10^{-9}(-50\sqrt{3})(10^{-3}) = \boxed{100\sqrt{3}\,PJ}$

e) of $(a_x + a_y + a_z)$?

$dw_{xyz} = -qE \cdot dL(a_x + a_y + a_z)$.

$= \frac{(10 + 50 + 100\sqrt{3})}{\sqrt{3}} = \boxed{135\,PJ}$

5

QNo 3:-

Solution:- a) $P(1,2,3)$ Forward $Q(2,1,4)$

The vector along this direction will be $Q-P=(1,-1,1)$ from which ~~$a_{PQ}=[a_x-a_y+a_z](2\times10^{-3})/\sqrt{3}$~~

$$a_{PQ}=[a_x-a_y+a_z]/\sqrt{3}$$

$$dW=-qE\cdot dL$$

$$=-(50\times10^{-6})\left[120a_\rho\cdot\frac{(a_x-a_y+a_z)}{\sqrt{3}}\right](2\times10^{-3})$$

$$=-(50\times10^{-6})(120[(a_x\cdot a_\rho)-(a_\rho\cdot a_y)])\frac{1}{\sqrt{3}}(2\times10^{-3})$$

At $P$, $\Phi=\tan^{-1}(2/1)=63.4^\circ$, Thus $(a_\rho\cdot a_x)=\cos(63.4^\circ)$

$$=0.447$$

$(a_\rho\cdot a_y)=\sin(63.4)=0.894$

Substituting these, we obtain

$$dW=3.1\,\mu J.$$

b) $Q=(2,1,4)$ Toward $P(1,2,3)$ A little Thought is in order here: Note that the field has only

(6)

a radical component and does not depend on $\Phi$ or $z$. And $P$ and $Q$ are at the same radius ($\sqrt{5}$) from $z$ axis. Thus the answer is $dW = 3.14\mu J$ as in part a. This is also found by going through the procedure as in part a. but with the direction (Roles of $P$ & $Q$) reversed.

**Q No 9:- compute the value $\int_A^P G \cdot dL$.**

Solution:-

a) Straight line segments $A(1,-1,2)$ to $B(1,1,2)$ to $P(2,1,2)$: In general we would have.

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx.$$

The change in $x$ occurs when moving b/w $B$ & $P$, during which $y = 1$.

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx$$

$$\int_1^2 2(1)\,dx.$$

$$= \frac{2}{2}$$

b) straight line segment A(1,-1,2) to C(2,-1,2) to P(2,1,2), In this case change in $x$ occurs when moving from A to C during which $y=-1$

$$\int_A^P G\cdot dL = \int_C^P 2y\,dx$$

$$= \int_1^2 2(-1)\,dx$$

$$= \underline{\underline{-2}}$$

**Q No 5: let $G = 3xy^3 a_x + 2z a_y$. Find**

Solution:- let $G = 3xy^3 a_x + 2z a_y$

a) straight line $y = x-1$, $z=1$

$$= \int G\cdot dL = \int_2^4 3xy^2 + \int_1^3 2z\,dy$$

$$= \int_2^4 3x(x-1)^2\,dx + \int_1^3 2(1)\,dy$$

$$= \boxed{90}$$

b) parabola $6y = x^2+2$, $z=1$

$$\int G\cdot dL = \int_2^4 3xy^2 + \int_1^3 2z\,dy$$

⑧

$$= \int_2^4 \frac{1}{12} x (x^2+2)^2 + \int_1^3 2(1)\, dy$$

$$= \boxed{82}$$