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Section "B"

Fourth Semester

Subject: Differential Equation

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Q No #01. Solve the following objective type Questions.

(i) The order of matrix A is $m \times p$ and order of B is $p \times n$.
Then the order of matrin AB is ?

Solution:- The order of A matrin is equal to the Number of its Rows Multiply by Number of its Column. So $A = m \times p$ where "m" number of Rows and "p". Number of Column.

Similarly $B = p \times n$ where "p" number of Rows and p numbers of Column. Now According to the Rule Number of Column in A is equal to the Numbers of Rows in B. Then we can multiply A And B That as

$$\boxed{AB = m \times n}$$

(ii) The Number of Non-zero rows in an Echelon form?

Sol:- The Numbe of non-zero rows in a Echelon form is of a Matrix is called "RANK".

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singuler matrix than $a = ?$

Solution: For Singuler matrix $|B| = 0$

So $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

& $|B| = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} = 1 \times a - 4 \times 2 = 0$

$a - 8 = 0$

$\boxed{a = 8}$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:- $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$= (2i)(-i) - (i)(i)$

$= -2i^2 - i^2$

we know that $i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$\boxed{|A| = 3}$

(V) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is $=$?

Sol:- If each element of a principle diagnal of a matrix is Some non-zero Scaler and all other elements are zero that it is a Scalar Matrix So,

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix.

(vi) Solution of $\frac{dy}{dx} + 2xy = y$ is ?

Solution: $\frac{dy}{dx} + 2xy = y$

$\Rightarrow \frac{dy}{dx} = y - 2xy$

$\Rightarrow \frac{dy}{dx} = y(1-2x)$

$\Rightarrow \frac{dy}{dx} = (1-2x)\,dx$

$\Rightarrow \int \frac{dy}{y} = \int 1\,dx - 2\int x\,dx$

$\Rightarrow \ln y = x - \frac{\cancel{2}x}{\cancel{2}} + c$

$\Rightarrow \ln y = x - x^2 + c$

(vii) The order and degree of differential equation:

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$ is ?

Sol:- The order of differential equation is the order of highest order of derivatives and degree of highest order derivatives.

order $= 1$, Degree $= 3$

(viii) The order and degree of differential $\frac{d^2y}{dx^2} - 4xy = \sin\frac{d^2y}{dx}$ is ?

Solution :.

So Order $= 2$

Degeree is Not defined.

(IX) The differential equation

$$2\frac{dy}{dx} + x^2 y = 2x+3 \quad , \quad y(0) = 5$$

is = ?

Sol:- $2dy + x^2 y = (2x+3)\,dx$

$$2dy = (2x+3-x^2y)\,dx$$

$$2\int dy = 2\int x\,dx + 3\int dx - y\int x^2 dx$$

$$2y = \frac{2x^2}{2} + 3x - \frac{yx^3}{3} + c$$

$$2y = x^2 + 3x - \frac{yx^3}{3} + c$$

Both side divided By 2

So we get

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c \text{ ---- } (A)$$

initial Conditin

$$y(0) = 5 \text{ So } x = 0, \ y = 5$$

Put in equation (A)

$$5 = \frac{(0)^2}{2} + \frac{3(0)}{2} - \frac{y(0)^3}{6} + c$$

$$5 = 0 + 0 - 0 + c$$

$$\boxed{C = 5}$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c$$

(X) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:-

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = |A|$$

Expand By R_1

$$|A| = +1 \begin{vmatrix} b & b^2 \\ c & c \end{vmatrix} - a \begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2 \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= +1(bc^2 - b^2c) - a(c^2 - b^2 + a^2(c - b)$$

$$|A| \Rightarrow bc^2 - b^2c - ac^2 + ab^2 + ac^2 - a^2b$$

Q No#02: Express the determinant

Part (i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

is The product of factors which are linear in a, b, c,

Solution:-

let $A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

$|A| \; a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$

$|A| = a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$

$|A| = ab^2c^3 - ab^3c^2 - ba^2c^3 + ba^3c^2 + ca^2b^3 - ca^3b^2$

Q No#02

Part (ii)

Find Eigen Value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Solution:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equation $\Rightarrow |A-\lambda I| = 0 \cdots (A)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we take the Determinante.

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand By R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$- \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \text{ ---- } B$$

Again Expand By R_1

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1\left[(-1)(2-\lambda)\right.$$
$$\left. - (-1)(-1)\right] - 1\left[(-1)(-1) - (-1)(3-\lambda)\right]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-3+\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \quad - (a)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand By C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2 + \lambda - 1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \ldots\ldots b$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by Column 1

$$\Rightarrow -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2 + \lambda - 1) + 1(6 - 3\lambda - 2\lambda + \lambda^2 - 1)\right]$$

$$-(3 - \lambda + \lambda^2 - 5\lambda + 5)$$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8}$ ..... (c)

Put eq (a), (b), and (c) in (B)

$\Rightarrow (2-\lambda)[-\lambda + 8\lambda^2 - 18\lambda + 8] - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8$

$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2$

$+ 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda$

$+ 6\lambda + 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By synthatic Division

we get:

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \quad \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

$$\lambda^2 - 8\lambda + 16 = 0$$

By factorization Method.

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4)$$

$$(\lambda - 4)(\lambda - 4)$$

$$\lambda = 4, \lambda = 4$$

So $\boxed{\lambda_1 = 0 \quad \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4}$

QNo#03 The rate of change in the form of differential equation is given by

$$(x^2 + 3y)\,dx - 2xy\,dy = 0$$

Find Genral Solution at

$$x = 2 \;\&\; y = 6$$

Solution:-

$$\Rightarrow (x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2,\ y = 6$$

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

Dividing Both side By $2xy\ dx$

we get

$$\frac{dy}{dx} = \frac{x^2+3y}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right]$$

Let $y = vx$

Differential

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{— (9)}$$

So $v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$

Multiplying Both side By "2"

$$\Rightarrow 2v + 2x\frac{dv}{dx} = \left[\frac{1}{v} + 3v\right]$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$\Rightarrow$ By taking l.c.m

$$\Rightarrow 2x\frac{dv}{dx} = v * \frac{1}{v} + \frac{3v}{1} * v - \frac{2v}{1} * v$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1 + 3v^2 - 2v^2}{v}$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1 + v^2}{v}$$

$$\Rightarrow 2x\,dv = \left(\frac{1 + v^2}{v}\right) dx$$

$$\Rightarrow \frac{v}{1 + v^2}\,dv = \frac{dx}{2x}$$

$$\Rightarrow \int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \ln|1+v^2| = \ln x + \ln c$$

$$\cancel{\ln}|1+v^2| = \cancel{\ln}\, xc$$

$$1+v^2 = xc$$

put $v = \frac{y}{x}$

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3 c \text{ ...... (11)}$$

Put $x=2,\ y=6$ in eq (11)

$$4+36 = 8c$$

$$c = \frac{\cancel{40}\,5}{\cancel{8}}$$

$$\boxed{c = 5}$$

$\boxed{C=5}$ — put in eq (ii)

$$x^2+y^2=5x^3$$

$$y^2=5x^3-x^2$$

$$y^2=x^2(5x-1)$$

Taking square Root on Both side.

$$y=+x\sqrt{5x-1}\ ,\ y=-x\sqrt{5x-1}$$

$$\boxed{y=\pm x\sqrt{5x-1}}$$