P. No 1

Department of Electrical Engineering

Assignment

25/06/2020

Course details

Course title :- Signal and System
Module :- 4th
Instructor :- Engr Mujtaba Ihsan
Total Marks :- 50

Student Details

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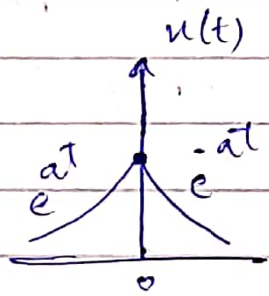
Q:-5 Apply fourier transform on the signal, $x(t) = e^{-a|t|} \cdot u(t)$ where $u(t)$ is a unit step function.

Sol:- The Fourier transform of the given function $x(t)$ is given by:

$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-jwt}\, dt$$



Note:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$$\therefore X(jw) = \int_{-\infty}^{0} e^{at}\, e^{-jwt}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-jwt}\, dt$$

$$X(jw) = \int_{-\infty}^{0} e^{(a-jw)t}\, dt + \int_{0}^{\infty} e^{-(a+jw)t}\, dt$$

$$= \frac{e^{(-a-jw)t}}{a-jw}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+jw)t}}{-(a+jw)}\Bigg|_{0}^{\infty}$$

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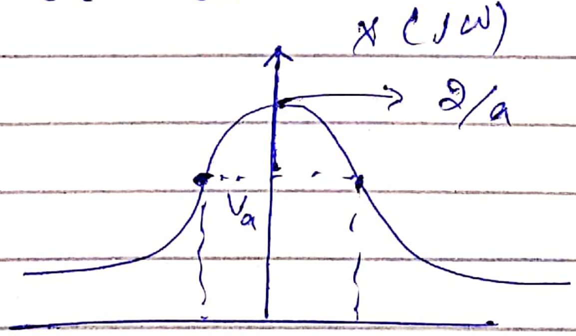
$$= \frac{1}{(a-jw)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+jw)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-jw)}[1-0] - \frac{1}{(a+jw)}[0-1]$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw}$$

$$= \frac{a+\cancel{jw}+a-\cancel{jw}}{a^2-(jw)^2}$$

$$x(jw) = \frac{2a}{a^2-w^2}$$



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Q:-3 If $X(z) = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$ Retrieve $x[n]$ using inverse z-transform.

Sol:-

$$X[z] = \frac{2z^2 - 2z}{z^2 + 2z - 3}$$

$$x[z] = \frac{2z(z+1)}{z^2 + 3z - z - 3}$$

$$x[z] = \frac{2z(z+1)}{z(z+3) - 1(z+3)}$$

$$\frac{X[z]}{z} = \frac{2[z+1]}{(z+3)(z-1)}$$

By partial fraction.

$$\frac{2(z+1)}{z^2 + 2z - 3} = \frac{A}{z+3} + \frac{B}{z-1} \rightarrow (2)$$

$$2(z+1) = A(z-1) + B(z+3) \rightarrow (1)$$

put $z = 1$ in (1)

$$2(1+1) = A(1-1) + B(1+3)$$

$$\cancel{4} = \cancel{4}B$$

$$\boxed{B = 1}$$

put $z = -3$ in eq ①

$2(-3+1) = A(-3-1)$

$\cancel{-4} = \cancel{-4}A$

$\boxed{A = 1}$

Now put A, B in eq ②

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$X[z) = \frac{z}{z+3} + \frac{z}{z-1}$$

inverse z-transform.

$x\{n] = 1u[n] + 1(3)^k$

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~~So the inverse Z-transform is~~

~~$x[n] = \frac{1}{2} u[n] + \frac{1}{2}(3)^n$.~~

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Q:- 1 (a) Show with the help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.

Sol:- We know that differentiation in time domain corresponds to multiplication by jw in frequency domain. From the property, we might suspect that multiplication by jt in the time domain corresponds roughly to differentiation in frequency domain.
As we know that,

$$X(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

Differentiating both sides w.r.t "w"

$$\frac{d}{dw} X(jw) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-jwt}\, dt$$

$$\frac{d}{dw} X(jw) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$\frac{d}{dw} X(jw) = -jt\ \mathcal{F}\{x(t)\}$$

2) $-jt\, x(t) \xleftrightarrow{\mathcal{F}} \frac{d}{dt} X(jw)$

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(b) If

$x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

produce $Y[z]$ and $y[n]$.

Sol:- $X[z] = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + z^{-1} + 2z^{-2}$

Now

$Y(z) = H(z) \times X[z]$

$= (3 + z^{-1} + 2z^{-2}) \times (2 - 4z^{-2} + 2z^{-3})$

$Y(z) = 6 - 12z^{-2} - 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4}$
$+ 4z^{-2} - 8z^{-4} + 4z^{-5}$

$y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

To find $y[n]$ use the dely property.

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3]$
$- 6\delta[n-4] + 4\delta[n-5]$.

Q:-2 $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$

Retrieve the Fourier series for the given function.

Sol:- As

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi}\frac{\pi}{2}\,dx$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} 1\,dx + \frac{\pi}{2}\int_{0}^{\pi} 1\,dx\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}\,x\Big|_{-\pi}^{0} + \frac{\pi}{2}\,x\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}\left[0-(-\pi)\right) + \frac{\pi}{2}\left[\pi-0\right]\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}(\pi) + \frac{\pi}{2}(\pi)\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{\pi^2}{2} + \frac{\pi^2}{2}\right]$$

$$a_0 = \frac{1}{2\pi}(0)$$

$$\boxed{a_0 = 0}$$

Now,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\frac{\sin nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\sin n(0) - \sin n(-\pi)\right] +$$

$$\frac{\pi}{2}\left[\sin n(\pi) - \sin n(0)\right].$$

$$a_n = \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$a_n = \frac{1}{n\pi}(0)$$

$$\boxed{a_n = 0}$$

Now,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x) \sin nx\, dx + \int_{0}^{\pi} f(x) \sin nx\, dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2} \sin nx\, dx + \int_{0}^{\pi} \frac{\pi}{2} \sin nx\, dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} \, -\frac{\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2} \, -\frac{\cos nx}{n}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-\cos n(0) + \cos n(-\pi)\right) + \frac{\pi}{2}\left[-\cos n(\pi) + \cos n(0)\right]\right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-1 + \cos n(-\pi)\right] + \frac{\pi}{2}\left[-\cos n\pi + \cos n(0)\right]\right]$$

$$= \frac{\pi}{2} \cdot \frac{1}{n\pi}\left[-1\left[\cancel{+}1 + \cos n(-\pi)\right] + 1\left\{-\cos n\pi + 1\right\}\right]$$

$$= \frac{1}{2n}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd} \end{cases}$$

$$\boxed{b_n = \frac{4}{2n}}$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots\cdots +$$
$$b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots\cdots$$

$$f(x) = 0 + 0(\cos x) + 0(\cos 2x) + 0(\cos 3x) + \cdots\cdots$$

$$= \frac{4}{2}\sin x + 0 \sin 2x + \frac{4}{3(2)}\sin 3x + \cdots$$

$$= \frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \cdots\cdots$$

Q:-4 Express the transfer function using the given data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [0].$$

Sol:-

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1}B + D$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1}$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} s+2 & -1 \\ -1 & s \end{bmatrix}^{-1}$$

$$adj = (s+2)s + 1 = s^2 + 2s + 1$$

$$H[s] = \begin{bmatrix} 1, & 2 \end{bmatrix} \times \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2 + 2s + 1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+2s+1}$$

$$H[s] = \frac{s+2}{s^2+2s+1}$$

$$s^2+2s+1$$
$$s^2+s+s+1$$
$$s(s+1)+1(s+1)$$
$$(s+1)(s+1)$$