SULEMAN HAKEEM

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I.D= 16757

QUESTION # 1

Q $x^3y'' + 2x^2y' + 2y = 10x + \frac{10}{4}$

SOLUTION:-

$$\Rightarrow \left(x^3\frac{d^3}{dx^3} + 2x^2\frac{d}{dx} + 2\right)y = 10x + \frac{10}{x}$$

$$\Rightarrow (x^3D^3 + 2x^2D^2 + 2)y = 10x + \frac{10}{x} \quad \text{— (A)}$$

put $x^3D^3 = \Delta$

$$\Rightarrow x^2D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 + 3\Delta^2 + 2\Delta$$

and $x = e^t$

putting in eq (A).

$$\Rightarrow (\Delta^3 - 3\Delta^2 + 2\Delta + 2D^2 - 2\Delta + 2)y = 10e^t + \frac{10}{e^t}$$

$$\Rightarrow (\Delta^3 - \Delta^2 + 2)y = 10e^t + \frac{10}{e^t}$$

The characteristic equation is $\Delta^3 - \Delta^2 + 2 = 0$. Now By using Synthetic division to find its root.

	1	-1	0	2
-1		-1	2	-2
	1	-2	2	0

$$\Delta^2 - 2\Delta + 2 = 0$$

Now using Quadratic equation

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$$\Delta = \frac{-b \pm \sqrt{b^2+4ac}}{2a}.$$

$$\Delta = \frac{-(-2) \pm \sqrt{4-8}}{2}$$

$$\Delta \quad \frac{2 \pm \sqrt{4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{2^2} \times 2}{2}$$

$$\Delta = \frac{2 \pm 2i}{2}$$

$$\Delta = \frac{\cancel{2}(1 \pm 1)}{\cancel{2}}$$

$$\Delta = 1 \pm i$$

Since the root are complex.

$$y_c = e^{-x}(c_1 \cos t + c_2 \sin t)$$

For y_p.

$$y_p = \frac{1}{D^3 - D^2 + 2} 10e^t + \frac{1}{D^3 - D^2 + 2} \frac{10}{e^t}$$

$$\Rightarrow \frac{10e^t}{x - x + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$\Rightarrow \frac{\overset{5}{\cancel{10}}e^t}{\cancel{2}1} + \frac{\overset{5}{\cancel{10}}e^{-t}}{1 - 1 + \cancel{2}1}$$

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$y_p = 5e^{t} + 5e^{-t}$

Now general Solution is.

$y = y_c + y_p$.

$y = e^{-t}(c_1 \cos t + c_2 \sin t) + 5e^{t} + 5e^{-t}$.

put $e^{t} = x$ and $t = \log x$.

$$\boxed{y = e^{-x}(c_1 \cos \log x + c_2 \sin \log x) + 5e^{x} + 5e^{-x}}$$

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QUESTION # 5

$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2.$

SOLUTION:-

$\Rightarrow (x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$

e^{-x} $\Rightarrow ((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4)y = x^2.$

$\Rightarrow [(x+1)^2 D^2 - 3(x+1)D + 4]y = x^2$ — (A)

put $(x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

$x = e^t$

put in eq (A)

$\Rightarrow [\Delta^2 - \Delta - 3D + 4]y = e^{2t}.$

$\Rightarrow (D^2 + 4D + 4)y = e^{2t}.$

$\Rightarrow (D^2 + 4D + 4)y = e^{2t}.$

For y_c we find the root By factorization

$\Delta^2 - 4\Delta + 4 = 0$

$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$

$\Delta(\Delta - 2) - 2(\Delta - 2) = 0.$

$\Delta - 2 = 0$, $\Delta = 2.$

$\Delta - 2 = 0$, $\Delta = 2$

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So the root are real and Repeat.

The general Solution as.

$$mx$$

$$y = (c_1 + c_2 x)$$

$$y = (c_1 + c_2 x)^{2x}$$

for $y_p = ?$

$$(2)^2 - 4(2) + 4 = 0.$$

$$y_p = \frac{1}{D^2 - 4D + 4} e^{2t}.$$

So we take derivative.

$$y_p = \frac{2}{2D - 4} e^{2t}.$$

So if put 2.

$$2D - 4 \Rightarrow 2(2) - 4 = 0.$$

This again goes to 0.

So we take again derivative.

$$y_p = \frac{\cancel{2}}{\cancel{2}} e^{2t}$$

The general Solution is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x)^{2t} + e^{2t}.$$

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QUESTI

$$x^3 \frac{d^3y}{dx^3}$$

SOLUTIO

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QUESTION :- 2

$$x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 15y = x^4.$$

SOLUTION :-

let $D = \frac{dy}{dx}$.

$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$ — (1)

Let $x = e^t \Rightarrow t = \ln x$.

$xD = \Delta$

$x^2D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$.

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta + 2\Delta$.

Substitution in eq (1).

$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5(\Delta) - 15)y = e^{4t}$

$(\Delta^3 = \Delta^2 - 7\Delta - 15)y = e^{4t}$.

Using Synthetic division.

	1	+1	−7	−15
5		5	12	15
	1	4	5	0

$\Delta^2 + 4\Delta + 5 = 0$

Using Quadratic formula

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$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm \sqrt{4}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$\Delta = \frac{2(-4 \pm i)e}{i}$$

$$\Delta = -2 + i$$

$$\Rightarrow y_c = e^{xt}(c_1 \cos t + c_2 \sin t)$$

and

$$y_p = \frac{1}{D^3 + D^2 - 7D - 15} e^{4t}$$

$$= \frac{1}{4^3 + 4^2 - 7(4) - 15} e^{4t} \quad \because D = 4$$

$$= \frac{1}{37} e^{4t}$$

So

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

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Put

$$y = e^{3x}$$

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Put $t = \ln x$ and $x = e^{4t}$.

$y = e^{3x}(C_1 \cos \log x + C_2 \sin \log x) + \frac{1}{37} e^{4x}$

Question: 3

$x^2y'' + 2xy' - 6y = 10x^2$

SOLUTION:

$y(1) = 1$ and $y'(1) = -6$

$\left(x^2\frac{d^2y}{dx^2} + 2x\frac{d}{dx} - 6\right)y = 10x^2$

put $xD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$x = et$ and $\log x = t$

$(D^2 - D + 2D - 6)y = 10e^{2t}$

$(D^2 + D - 6)y = 10e^{2t}$

The characteristic equation

$D^2 + D - 6 = 0$

$D^2 + 3D - 3D - 6 = 0$

$= D(D+3) - d(D+3) = 0$

$(D+3)(D-2) = 0$

$D+3 = 0$, $D-2 = 0$

$D = 2$, $D = -3$

Since roots are real and distinct

For $y_c = ?$

$y_c = C_1e^{-3t} + C_2e^{2t}$

$\therefore D = 4$

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For $y_p = ?$

$$y_p = \frac{1}{D^2 - D - 6} e^{2t}$$

$$\Rightarrow \frac{10}{D^2 - D - 6} e^{2t}$$

$$= 10 \cdot \frac{1}{0} e^{2t}$$

$$10 \frac{1}{d/dD (D^2 + D - 6)} e^{2t}$$

$$\Rightarrow 10 \frac{t}{2D + 1} e^{2t}$$

$$10 \frac{1 \cdot t}{4 + 1} e^{2t}$$

$$y_p = 2 t e^{2t}$$

general Solution:

$$y = y_c + y_p$$

$$= C_1 e^{-3t} + C_2 e^{2t} + 2 e^{2t}$$

$$\Rightarrow y = C_1 x^{-3} + C_2 x^2 + 2 (\log x) x^2 \text{ ------ } *$$

put $y(1) = 1$ i-e $x = 1$, $y = 1$ put in $*$

$$1 = C_1 (1)^{-3} + C_2 (1)^2 + 2 \log(1)$$

$$1 = C_1 + C_2 \text{ ------ } *_1$$

Differentiate eq $*$ w.r.t x

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$y = -3c_1 x^{-4} + 2c_2 x + 2(x^2) + 4x \log x$

put $y(1) = 6$, $y' = -6$ and $x = -6$

$-6 = -3c_1 + 2c_2 + 2 + 0$

$-6 = -3c_1 + 2c_2 + 2$

$-6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2$ ---- a)

Multiplying eq ① with ② and dividing it from eq(a)

$$\begin{aligned} 2c_1 + 2c_2 &= 2 \\ -3c_1 + 2c_2 &= -8 \\ \hline 5c_1 &= 10 \end{aligned}$$

$c_1 = \frac{10}{5} = 2$

$c_1 = 2$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8 + 6$

$2c_2 = -2$

$c_2 = \frac{-2}{2} = -1$

$c_2 = -1$

Putting the value of c_1 and c_2 in *,

$y = 2x^{-3} - x^2 + 2\ln x (x^2)$

$y = 2/x^3 - x^2 + 2x^2 \log x$ answer.

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QUESTION # 4.

$$x^2y'' + 7xy' + 5y = \cancel{0}\, x^5$$

$$y(0) = 2 \quad , \quad y(1) = 2$$

SOLUTION :-

$$x^2y'' + 7xy' + 5y = x^5$$

$$x^2 \frac{dy^2}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5 \quad \text{---} \quad (1)$$

$$\left(x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5y\right) = x^5$$

Put $xD = \Delta$ and $x^2D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

$x = et \Rightarrow \log x = t$ in eq (1).

$$(\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$$

$$(\Delta^2 + 6\Delta + 5)y = e^{5t}$$

By Quadratic formula.

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-b \pm \sqrt{6^2 - 4(1)5}}{2(1)}$$

$$D = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm \sqrt{4^2}}{2} \Rightarrow \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$$D = -3 \pm 2$$

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Hence roots are real and distinct.

$y_c = C_1 e^{-5t} + C_2 e^{-t}$.

For $y_p = ?$

$$y = \frac{1}{D^2+6D+5} e^{5t}$$

$$= \frac{1}{5^2+6(5)+5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now general solution is.

$$y = y_c + y_p$$

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}.$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5 \quad \text{---- (2)}$$

$x = 0$ put in this equation.

Put $y(0) = 2$ i-e $y = 2$, $x = 2$

$$2 = C_1 (2)^{-5} + C_2 (2)^{-1} + \frac{1}{60} (2)^5$$

$$2 = -32C_1 - 2C_2 + \frac{1}{\cancel{60}_{15}} (\cancel{32}^{8})$$

$$2 = -32C_1 - 2C_2 + \frac{8}{15}$$

$$2 = \frac{-8}{15} = -32C_1 - 2C_2.$$

$$\frac{22}{15} = -32C_1 - 2C_2 \quad \text{---- (3)}$$

(1).

$= D^2 - D$

(1).

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Now differentiate eq (2) w.r.t. x.

$y' = -5C_1 x^{-6} - C_2 x^{-2} + \frac{1}{12} x^4$

put $y'(2) = 2$ ie $y' = 2$ and $x = 2$.
in above equation.

$2 = -5C_1 (2)^{-6} - C_2 (2) + \frac{1}{12} (2)^4$.

$2 = -5C_1 (-64) - C_2 (4) + \frac{1}{\cancel{12}\,3} (\cancel{16})^4$.

$2 = 320C_1 + 4C_2 + \frac{4}{3}$

$2 - \frac{4}{3} = 320C_1 + 4C_2$ ---- (4).

Multiplying equation (3) with (2).
and than from (4).

$\frac{-44}{15} = 64C_1 + 4C_2$

$* \quad \frac{-44}{15} = 64C_1 + 4C_1$

$\frac{2/3}{} = \pm 320C_1 \pm 4C_2$

$\frac{34}{15} = -256C_1$

$C_1 = \frac{34}{15} \times 256$

$C_1 = 580$

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Put
$\frac{22}{15}$

$\Rightarrow \frac{188}{-2}$

Now
C_2 in

$y = 5$

$y =$

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Put this value in eq (3)

$$\frac{22}{15} = -326{,}580 - 2C_2$$

$$\frac{-138}{-2} = C_2 \Rightarrow C_2 = -9280$$

Now put the value of C_1 and C_2 in eq (2)

$$y = 580x^{-5} - 9820x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9820}{x} + \frac{1}{60}x^5$$ Answer