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Dept :- BE(Electrical)
Subject :- DSP

Q1:
i)-
Ans:- As we are given :
Consider the following analog signals

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

i)- Determine minimum sampling rate required to avoid aliasing.

$$f_s \geq 2f_{max}$$

$$\because f = \frac{\omega}{2\pi}$$

$$f_1 = \frac{100\pi}{2\pi}, \quad f_2 = \frac{200\pi}{2\pi}.$$

$$f_1 = 50Hz, \quad f_2 = 100Hz$$

So:-
f_2 is max (greater ratio)
$(f_s \geq 2 \times 100\ Hz)$
sample frequency to avoid aliasing.

ii)-
$F_s = 100Hz$
f_1 becomes

$$f_1' = f_1/F_s = \frac{5\cancel{0}}{10\cancel{0}} = (0.5Hz)$$

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f_2 becomes

$$f_2 = f_2/100 = 100/100 = 1\,Hz$$

So:-

$$\omega_1' = 2\pi f_1' \quad , \quad \omega_2' = 2\pi f_2'$$
$$\omega_1' = 2\pi \times 0.5 \quad , \quad \omega_2' = 2\pi \times 1$$
$$\omega_1' = \pi \quad , \quad \omega_2' = 2\pi .$$

$$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$$

The signals are

$$x[n] = 3\cos \pi n + 4\sin 2\pi n$$

This effect of sampling rate on the newly generated discrete time signal is that their will be no aliaising phenemona. There will not present unwanted component in the reconstruction of the signal.

$$\omega_1 = 100\pi \qquad f_r = 200\pi$$

$$f_1 = \frac{\cancel{100\pi}}{\cancel{2\pi}} \quad , \quad f_r = \cancel{100\pi}$$

$$(f_1 = 50)$$

iii)-

folding frequency of the sampled signal is:

$$\text{folding "f"} = F_s/2 = 100/2$$
$$= 50\,Hz$$

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we have frequency of the orignal signals

$$f_1 = 50Hz \; , \; f_2 = 100Hz$$

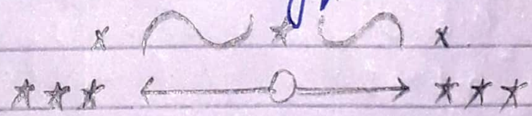
both the frequency are either equal or greater than folding frequency.

Hence for ideal interpolation we can construct the orignal signal.

$$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t \, .$$

The orignal signal is constructed because we can and we use sampling frequency at Nyquist rate

we can also reconstruct the signal for sampling frequency above the nyquist rate.



Q1:-

b)- Consider a discrete time signal which is given by:-

$$x(n) = \begin{cases} 0.5^n \, , & n \geq 0 \\ 0 \, , & n < 0 \end{cases}$$

i)- Draw the sampled signal:-

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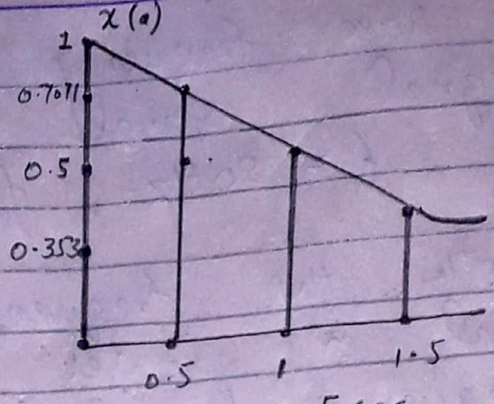
x_n	0.5
0	1
0.5	0.7071
1	0.5
1.5	0.353



$T = 0.5\ sec$

ii)-
Sol:-

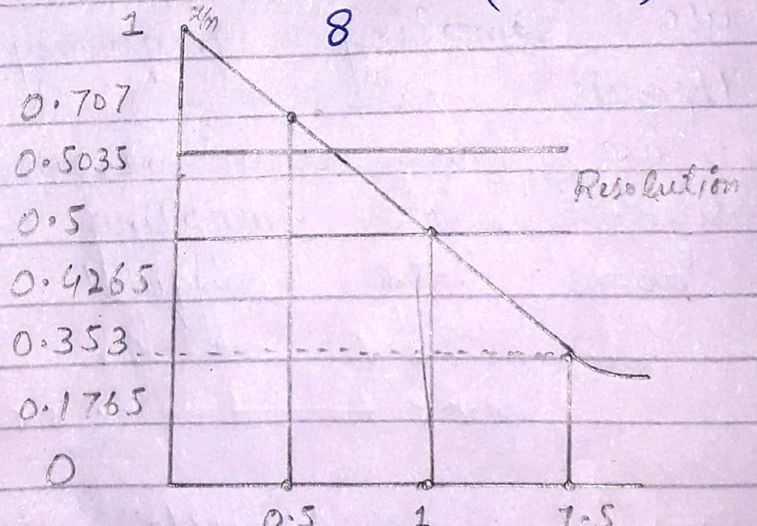
$L = 2^n$

$n = bits = 3$

$L = 2^3 = 8$ levels.

$$Resolution = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = (0.125)$$



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	Discrete Time Signal	Truncation	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

Q2:-

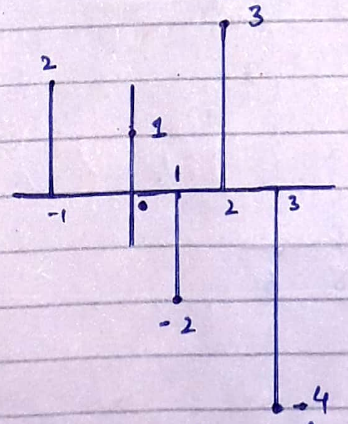
a) Determine the response of the system to the following input signal with given impulse response.

$x[n] = \{2, 1, -2, 3, -4\}$

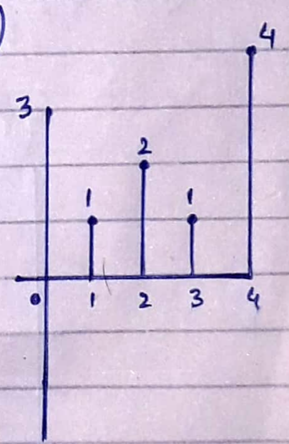
$h[n] = \{\overset{3}{\uparrow}, 1, 2, 1, 4\}$.

Sol:-

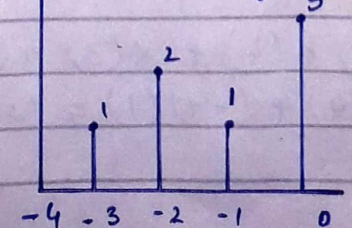
$$x[n] = \sum_{100\infty}^{\infty} x[K]\,h(n-k)$$



h(k)



h(-k) folded signal



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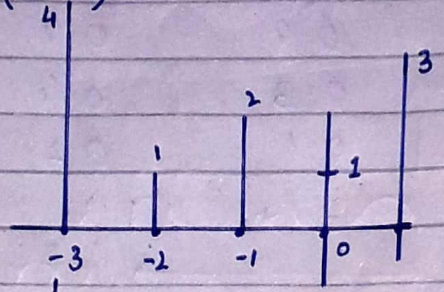
$$Y[0] = \sum_{k=1}^{0} x(-1)h(-1) + x(0)h(0)$$

$$Y(0) = (2)(1) + (1)(3)$$

$$\Rightarrow 2 + 3 = 5$$

for $n = 1$

$h(1-k)$



$$Y(1) = \sum_{k=-1}^{1} x(n)\, h(1-k)$$

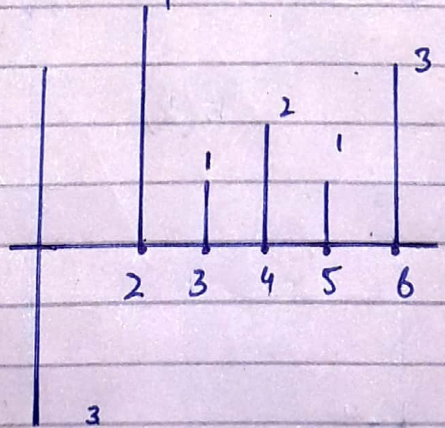
$$= x(-1)h(-1) + x(0)h(0) + 0\, x(1)h(1) + x(h(1)) + x(2)x(2) + 0(x(3)h(3))$$

$$Y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3).$$

$$= 8 + 1 - 4 + 3 - 12 = -4$$

$(n = 3)$

$h(3-k)$



$$Y(3) = \sum_{k=2}^{3} x(x)\, h(n-k)$$

$$= x(2)h(2) + x(3)h(3)$$

$$= (3)(4) + (-4)(1) = 12 - 4 = 8$$

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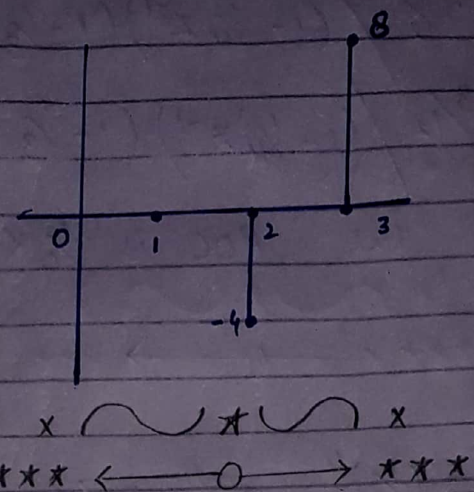

Q3:-
i)-

$$x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3), & n < 0 \end{cases}$$

Sol:-

$$x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0. \end{cases}$$

writing in the form of z-transform

$$X(z) = \sum_{n=0}^{\infty} (1/4)^n_z + \sum_{n=-\infty}^{0} (1/3)^n z^n - 1.$$

using geomatric series:

$$= \frac{1}{1 - 1/4 z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1.$$

$$= 1/1 - 1/4 z^{-1} + 1/1 - 1/3 - 1 - 1$$

$$= 1 - 1/1/4 z^{-1} + 1 - 1/1 - 1 \,/\, (1 - 1/4 z^{-1})(1 - 1/3 z^{-1})$$

$$= \frac{1 - 1/3 z + 1 - 1/4 z^{-1} - (1 - 1/4 z^{-1})(1 - 1/3 z)}{(1 - 1/4 z^{-1})(1 - 1/3 z)}$$

$$= \frac{1 - 1/3 z + 1 - 1/4 z^{-1} - (1 + 1/3 z - 1/4 z^{-1} + 1/12 z^{-1} z)}{(1 - 1/4 z^{-1})(1 - 1/3 z).}$$

$$= \frac{1 - \cancel{1/3} z + 1 - \cancel{1/4} z^3 - 1 + \cancel{1/3} + 1/4 \cancel{z^3} + 1/12}{(1 - 1/4 z^{-1})(1 - 1/3 z)}$$

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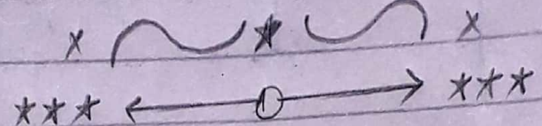
$$= 1 + \frac{1}{12} \Big/ (1 - \tfrac{1}{4} z^{-1})(1 - \tfrac{1}{3} z).$$

$$= \frac{13}{12} \Big/ (1 - \tfrac{1}{4} z^{-1})(1 - \tfrac{1}{3} z).$$

Hence the ROC is $\frac{1}{4} < |z| < 3$.



Q3:-
ii)-
Sol:-

$$x(n) = \begin{cases} (\frac{1}{2})^4, \ -3^n, & n \geq 0 \\ 0, & \text{else where.} \end{cases}$$

In the form of z- transform;

$$X(z) = \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n} z^{-n} - \sum_{n=0}^{\infty} 0\, 3^n z^{-n}.$$

using geomatric series to somplify it.

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{1 - 3z^{-1} - 1 + \frac{1}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3z^{-1})}$$

$$= -\frac{5}{2} z^{-1} \Big/ (1 - \tfrac{1}{2} z^{-1})(1 - 3z^{-1})$$

The ROC is
$|z| > 3$