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Q

NAME	JIBRAN
ID	14472
SEMESTER	4^{th}
SUBJECT	DIFFERENTIAL EQUATION
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Q (1) part (a)

Q Define 2^{nd} order linear homogeneous/ non-homogeneous equations along with two examples.

Homogeneous equation:-

For each equation we can write its related homogeneous or complementary equation $y''+py'+qy=0$

Non-Homogeneous equation:-

The nonhomogeneous differential equation of this type has the form $y''+py'+qy=f(x)$, where p, q are constant numbers (that can be both as real as complex numbers.

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Q (1)(b) part (1)

$16y'' + 24y' + 9y = 0$

Solution:-

let $y = e^{rx}$

$16r^2e^{rx} + 24re^{rx} + 9e^{rx} = 0$

$e^{rx}(16r^2 + 24r + 9) = 0$

$16r^2 + 24r + 9 = 0$

$16r^2 + 12r + 12r + 9 = 0$

$4r(4r+3) + 3(4r+3) = 0$

$(4r+3)(4r+3) = 0$

$r = -\frac{3}{4}$ repeated

$y = c_1e^{rx} + c_2xe^{rx}$

putting values.

$$\boxed{y = c_1e^{-\frac{3}{4}x} + c_2xe^{-\frac{3}{4}x}}$$ Ans.

Q (1) (b) part (2)

$$y'' - 4y' - 12y = 3e^{5t}$$

Solution

$$y'' - 4y' - 12y = 3e^{5t}$$

$$y'' - 4y' - 12y = 0$$

The characteristic equation for this differential equation and its roots are

$$r^2 - 4r - 12 = (r-6)(r+2) = 0$$

$$\Rightarrow r_1 = -2, \quad r_2 = 6$$

Complementary solution is then

$$y_c(t) = C_1 e^{-2t} + C_2 e^{6t}$$

$$Y_p(t) = Ae^{5t}$$

Plugging into the differential equation gives

$$25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t}) = 3e^{5t}$$

$$-7Ae^{5t} = 3e^{5t}$$

$$-7A = 3 \Rightarrow A = -\frac{3}{7}$$

$$\boxed{Y_p(t) = -\frac{3}{7}e^{5t}}$$ Ans

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Q(2) part (i)

$2y'' + 5y' + 3y = 0 \quad y(0) = 3 \quad y'(0) = -4$

Solution

let $y = e^{rt}$

$2r^2 e^{rt} + 5r e^{rt} + 3e^{rt} = 0$

$e^{rt}(2r^2 + 5r + 3) = 0$

$e^{rt}(2r^2 + 2r + 3r + 6) = 0$

$2r(r+2) + 3(r+2) = 0$

$(2r+3)(r+2) = 0$

$r_2 = -\frac{3}{2}, \quad r_1 = -2$

$\boxed{y = C_1 e^{-2t} + C_2 e^{-\frac{3}{2}t}}$ eq (A)

putting $y(0) = 3$

$3 = C_1 e^{-2(0)} + C_2 e^{-\frac{3}{2}(0)}$

$\boxed{3 = C_1 + C_2} \rightarrow \boxed{C_1 = 3 - C_2} \rightarrow (1)$

Taking derivative of eq (A)

$y' = C_1(-2)e^{-2t} + C_2\left(-\frac{3}{2}\right)e^{-\frac{3}{2}t}$

$y' = -2C_1 e^{-2t} - \frac{3}{2} C_2 e^{-\frac{3}{2}t}$

$-4 = -2C_1 e^{-2(0)} - \frac{3}{2} C_2 e^{-\frac{3}{2}(0)}$

$-4 = -2C_1 - \frac{3}{2} C_2$

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$$-4 = -2(3-C_2) - \frac{3}{2}C_2$$

$$-4 = -6 + 2C_2 - \frac{3}{2}C_2$$

$$-4+6 = 2C_2 - \frac{3}{2}C_2$$

$$2 = 2C_2 - \frac{3}{2}C_2$$

$$2 = \frac{4C_2 - 3C_2}{2}$$

$$4 = 4C_2 - 3C_2$$

$$\boxed{4 = C_2}$$

value of C_2 put in eq (1)

$$C_1 = 3-4$$

$$\boxed{C_1 = -1}$$

putting values of C_1 and C_2 in eq (A)

$$y = -1e^{-2t} + 4e^{-\frac{3}{2}t}$$ Ans

Q(2) part (ii)

$2y'' + 5y' - 3y = 0 \quad y(0) = 3 \quad y'(0) = 4$

Solution:

$$2y'' + 5y' - 3y = 0$$

$$2r^2 + 5r - 3 = 0$$

$$r = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)}$$

$$r = \frac{-5 \pm \sqrt{25+24}}{4} \Rightarrow \frac{-5 \pm \sqrt{49}}{4}$$

$$\frac{-5}{4}, \frac{\sqrt{49}}{4}$$

$$r_1 = -\frac{5}{4}, \quad r_2 = \frac{7}{4}$$

$$y = C_1 e^{rt} + C_2 e^{rt}$$

$$y = C_1 e^{-5/4 t} + C_2 e^{7/4 t}$$

$$3 = C_1 e^{-5/4(0)} + C_2 e^{7/4(0)}$$

$$3 = C_1 + C_2 \Rightarrow C_2 = 3 - C_1$$

$$y' = -\frac{5}{4} C_1 e^{-5/4 t} + \frac{7}{4} C_2 e^{7/4 t}$$

$$4 = -\frac{5}{4} C_1 e^{-5/4(0)} + \frac{7}{4} C_2 e^{7/4(0)}$$

$$4 = -\frac{5}{4} C_1 + \frac{7}{4} C_2$$

$$4 = -\frac{5}{4} C_1 + \frac{7}{4}(3 - C_1)$$

$$4 = -\frac{5}{4} C_1 + \frac{21}{4} - \frac{7}{4} C_1$$

$$4 - \frac{21}{4} = \frac{-5C_1 - 7C_1}{4}$$

$$\frac{16-21}{4} = \frac{-12C_1}{4}$$

$$\cancel{-}\frac{5}{4} = \cancel{-}3C_1 \rightarrow \boxed{C_1 = \frac{5}{12}}$$

$$C_2 = \frac{36-5}{12}$$

$$\boxed{C_2 = \frac{31}{12}}$$

$$y = \frac{5}{12} e^{-5/4 t} + \frac{31}{12} e^{7/4 t}$$

Q (2) part (iii)

$y'' - 4y' + 9y = 0 \qquad y(0) = 0 \quad y'(0) = -8$

Solution:

$$r^2 - 4r + 9 = 0$$

root of this equation $r_{1,2} = 2 \pm \sqrt{5}i$

The general solution to the differential equation is then

$$y(t) = C_1 e^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t)$$

$$0 = y(0) = C_1$$

$$y(t) = C_2 e^{2t} \sin(\sqrt{5}t)$$

$$y'(t) = 2C_2 e^{2t} \sin(\sqrt{5}t) + \sqrt{5}C_2 e^{2t} \cos(\sqrt{5}t)$$

Apply the second initial solution to the derivative to get

$$-8 = y'(0) = \sqrt{5}C_2 \Rightarrow C_2 = -\frac{8}{\sqrt{5}}$$

The actual solution is then

$$\boxed{y(t) = -\frac{8}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)}$$

Ans

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$\sqrt{49}$

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Q(3)

Define laplace transform?

The laplace transforms of $f(t)$, $F(s)$, can be defined as

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \, dt.$$

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Q(3) part (A)

Find the laplace transforms of the given functions.

1) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

Solution:-

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$ Ans.

2) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Solution:-

$$G(s) = 4\frac{s}{s^2+(4)^2} - 9\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$ Ans.

3) $h(t) = e^{3t} + \cos(6t) - 3e^{3t}\cos(6t)$

Solution:-

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

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$$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36} \quad Ans.$$

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Q(4) part (a)

$$y'' - 4y' = e^{3t}$$

$$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} = \mathcal{L}\{e^{3t}\}$$

$$s^2\Psi(s) - sy(0) - y'(0) - 4[s\Psi(s) - y(0)] = \frac{1}{s-3}$$

$$s^2\Psi(s) - s(0) - 0 - 4(s\Psi(s) - (0)] = \frac{1}{s-3}$$

$$s^2\Psi(s) - 4(s\Psi(s)] = \frac{1}{s-3}$$

$$-4[s\Psi(s)] = \frac{1}{s-3} - s^2\Psi(s)$$

$$s\Psi(s) = \frac{1}{-4s+12} - \frac{s^2\Psi(s)}{-4}$$

$$\mathcal{L}\{y\} = \frac{1}{-4s+12} + \frac{s^2\Psi(s)}{4}$$

Q(4) part (b)

$$y'' + 3y' + 2y = e^{-t}$$

$$\mathcal{L}\{y''\} + 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{e^{-t}\}$$

$$s^2\Psi(s) - sy(0) - y'(0) + 3[s\Psi(s) - y(0)]$$

$$+ 2\Psi(s) = \frac{1}{s+1}$$

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$$s^2 \Psi(s) - s(0) - (0) + 3\left[s\Psi(s) - 0 \right]$$

$$+ 2\Psi(s) = \frac{1}{s+1}$$

$$s^2\Psi(s) + 3\Psi(s) + 2\Psi(s) = \frac{1}{s+1}$$

$$2\Psi(s) = \frac{1}{s+1} - s^2\Psi(s) + 3s\Psi(s)$$

$$\Psi(s) = \frac{1}{2}\left[\frac{1}{s+1} - s^2\Psi(s) + 3s\Psi(s) \right]$$

$$\mathcal{L}\{y\} = \frac{1}{2}\left[\frac{1}{s+1} - s^2\Psi(s) + 3s\Psi(s) \right]$$

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