

Department of Electrical Engineering

Course Title: Electromagnetic Field

Module: 4th semester

Student Detail

Name : Muhammad Ahmad

Roll No :14563

QNO1:- The value of E at _____

$$\textcircled{a} G = 2a_x - 3a_y + 4a_z.$$

Solution:-

\textcircled{a} in the direction of $a_p =$ the incremental work is given by $dW = -qE \cdot dL$, where in this case $dL = dp a_p = 6 \times 10^6 a_p$. Thus.

$$dW = -(20 \times 10^6 \text{ C})(100 \text{ V/m})(6 \times 10^6 \text{ m})$$
$$= -12 \times 10^9 \text{ J}$$

\textcircled{b} in the direction of $a_j =$ In this case $dL = 2d\phi a_j = 6 \times 10^6$ and so,

$$dW = -(20 \times 10^6)(-200)(6 \times 10^6)$$
$$= 2.4 \times 10^8 \text{ J}$$
$$= \boxed{24 \text{ nJ}}$$

\textcircled{c} in the direction of $a_z =$ Here, $dL = dz a_z = 6 \times 10^6 a_z$.

$$dW = -(20 \times 10^6)(300)(6 \times 10^6)$$
$$= -3.6 \times 10^8 \text{ J}$$
$$= \boxed{-36 \text{ nJ}}$$

\textcircled{d} In the direction of E .

$$a_E = \frac{100a_p - 200a_j + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$
$$= 0.267a_p - 0.535a_j + 0.802a_z.$$

$\textcircled{1}$

Thus

$$dW = -(20 \times 10^{-6}) [1000\hat{x} - 2000\hat{y} + 8000\hat{z}] \cdot [0.267\hat{x} - 535\hat{y} + 0.802\hat{z}] \times 6 \times 10^6$$

$$= \boxed{-44.9 \text{ mJ}}$$

② In the direction of $\hat{c} = 20\hat{x} - 30\hat{y} + 40\hat{z}$.

$$a_c = \frac{20\hat{x} - 30\hat{y} + 40\hat{z}}{[2^2 + 3^2 + 4^2]^{1/2}}$$

$$= 0.371\hat{x} - 0.557\hat{y} + 0.743\hat{z}$$

So Now

$$dW = -(20 \times 10^{-6}) [1000\hat{x} - 2000\hat{y} + 8000\hat{z}] \cdot [0.371\hat{x} - 0.557\hat{y} + 0.743\hat{z}]$$

$$= (6 \times 10^6) [37.1(a_p \cdot a_x) - 557(a_p \cdot a_y) - 74.2(a_p \cdot a_z)] + 111.4$$

$$= (6 \times 10^6) [22.2.9] \quad (6 \times 10^6)$$

where $\hat{a}_p \cdot \hat{a}_x = \cos(40^\circ) = 0.766$

$$(\hat{a}_p \cdot \hat{a}_y) = \sin(40^\circ) = 0.643$$

$$(\hat{a}_p \cdot \hat{a}_z) = -\sin(40^\circ) = -0.643$$

Now substituting these results in.

$$dW = -(20 \times 10^{-6}) [28.4 - 35.8 + 47.7 + 85.3 + 22.9]$$

$$= \boxed{-41.8 \text{ mJ}}$$

②

Q No 2 Let $E = 10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$.

Solution

(a) $E_p = -10 [\sin(\pi/6) a_x + 5 \sin(\pi/6) a_y + 10 \cos(\pi/6) a_z]$
 $= -[5 a_x + 25 a_y + 50\sqrt{3} a_z]$

(b) $dW_x = -q E \cdot dL a_x$
 $= -2 \times 10^{-9} (-5)(10^{-3}) = 10^{-11} \text{ J}$
 $= \boxed{10 \text{ pJ}}$

(c) of a_y ?
 $dW_y = -q E \cdot dL a_y$
 $= -2 \times 10^{-9} (-25)(-10^{-3}) = 50^{-11} \text{ J}$
 $= \boxed{50 \text{ pJ}}$

(d) of a_z
 $dW_z = -q E \cdot dL a_z$
 $= -2 \times 10^{-9} (-50\sqrt{3})(10^{-3})$
 $= \boxed{100\sqrt{3} \text{ pJ}}$

(e) of $(a_x + a_y + a_z)$?

$dW_{xyz} = -q E \cdot dL \frac{(a_x + a_y + a_z)}{\sqrt{3}}$
 $= \frac{10 + 50 + 100\sqrt{3}}{\sqrt{3}}$
 $= \boxed{135 \text{ pJ}}$

(3)

QNO3

Solution:-

① P(1,2,3) toward Q(2,1,4)

The vector along this direction will be $Q-P = \underline{(1, -1, 1)}$
from which $a_{PQ} = \frac{[a_x - a_y + a_z]}{\sqrt{3}}$

$$\begin{aligned} dW &= -qE \cdot dL \\ &= -(50 \times 10^6) \left[120 a_p \cdot \frac{(a_x - a_y + a_z)}{\sqrt{3}} \right] (2 \times 10^{-3}) \\ &= -(50 \times 10^6) (120) \left[(a_p \cdot a_x) - (a_p \cdot a_y) \right] \frac{1}{\sqrt{3}} (2 \times 10^{-3}) \end{aligned}$$

$$\text{At P, } \phi = \tan^{-1}(2/1) = 63.4^\circ, \text{ thus } (a_p \cdot a_x) = \cos(63.4^\circ) \\ = 0.447$$

$$(a_p \cdot a_y) = \sin(63.4^\circ) = 0.894$$

Substituting these, we obtain

$$dW = 3.1 \mu\text{J}$$

② Q(2,1,4) toward P(1,2,3) A little thought is in order here: Note that the field has only a radial component and does not depend on ϕ or z . And P and Q are at the same radius (r=5) from z axis. Thus the answer is $dW = \underline{3.1 \mu\text{J}}$ as in part a. This is also found by going through the procedure as in part a, but with the direction (roles of P & Q) reversed.

④

QNO4 Compute the value $\int_A^P G \cdot dL$.

Solution :-

(a) Straight line segments $A(1, -1, 2)$ to $B(1, 1, 2)$ to $P(2, 1, 2)$: In general we would have.

$$\int_A^P G \cdot dL = \int_A^P 2y dx.$$

The change in x occurs when moving b/w B & P , during which $y=1$.

$$\int_A^P G \cdot dL = \int_B^P 2y dx$$

$$= \int_1^2 2(1) dx.$$

(b) Straight line segment $A(1, -1, 2)$ to $C(2, 1, 2)$ to $P(2, 1, 2)$: In this case change in x occurs when moving from A to C during which $y=-1$

$$= \int_A^P G \cdot dL = \int_A^C 2y dx$$

$$= \int_1^2 2(-1) dx$$

$$= \underline{\underline{-2}}$$

(5)

QND 5 Let $Q = 3xy^3ax + 2zay$. Find.

Solution:-

Let $Q = 3xy^3ax + 2zay$.

(a) straight line $y = x - 1, z = 1$

$$\begin{aligned} &= \int Q \cdot dL = \int_2^4 3xy^2 + \int_1^3 2z dy \\ &= \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1) dy \\ &= \underline{\underline{190}} \end{aligned}$$

(b) Parabola $6y = x^2 + 2, z = 1$

$$\begin{aligned} &= \int Q \cdot dL = \int_2^4 3xy^2 + \int_1^3 2z dy \\ &= \int_2^4 \frac{1}{12} x(x^2 + 2)^2 + \int_1^3 2(1) dy \\ &= \underline{\underline{182}} \end{aligned}$$

(c)