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Paper earthquake & dynamic

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Q 1

Ans

Given data

- $\Rightarrow$  Length of beam  $= L = 10'$
- $\Rightarrow$  Beam pulled in downward direction  $= \frac{1}{2}''$
- $\Rightarrow E = 29000 \text{ Ksi}$
- $\Rightarrow I = 150 \text{ in}^4$
- $\Rightarrow S_t = 7337 \text{ Lb}$
- $\rightarrow$  ignore self wt & damping effect

Required :-

- i) Natural time period
- ii) develop and solve equation of motion for vibration of free end  $= ?$
- iii) develop equation showing variation the equivalent static force with time  $= ?$
- iv) amplitude of equivalent static force  $= ?$
- v) Graph

Solution :-

The general E.O.M for SDOF system is

$$kx + C\dot{x} + m\ddot{x} = P(t)$$

undamped system ( $C=0$ ) undergoing free vibration  
 $P(t) = 0$

So general E.O.M become

$$Ku + m\ddot{u} = 0 \rightarrow (1)$$

$$K = 3EI$$

$$K = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$K = 7.55 \text{ k/w}$$

In order to eliminate chances of mistake during calculation. It is more appropriate to use fundamental unit like lb, ft or kg, m, sec

$$K = 7.55 \text{ k/w} = 90625 \text{ lb/ft}$$

$$m = \frac{7337 \text{ lb sec}^2}{32.2 \text{ ft}} = 227.8 \text{ slug}$$

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{90625}{227.8}}$$

$$\omega_n = 20.5$$

$$\ln = \frac{2\pi}{\omega_n} = \frac{2\pi}{20} = 0.306$$

Putting value in eq (i)

$$90625 + 227.8 \ddot{u} = 0$$

where  $K$  is  $\text{lb/ft}$  &  $m$  is in  $\text{lb} \frac{\text{sec}^2}{\text{ft}}$

General solution to E.O.M for undamped free vibration is

$$u(t) = u(0) \cos(\omega_n t) + \frac{u(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24} \text{ ft} \quad \& \quad \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(20.5t) + 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos 20.5t$$

Equivalent static force at any time 't' is

$$f_s(t) = -K u(t) = \frac{90625 \times \cos(20.5t)}{24}$$

$$f_s(t) =$$

$$f_s(t) = 3776 \cos(20t)$$

Amplitude of dynamic displacement  $u_0$  for undamped free vibration is

$$u_0 = \sqrt{[ (u(0))^2 + (u(0)/\omega u)^2 ]}$$

$$= \sqrt{ \left( \frac{1}{24} \right)^2 + 0 }$$

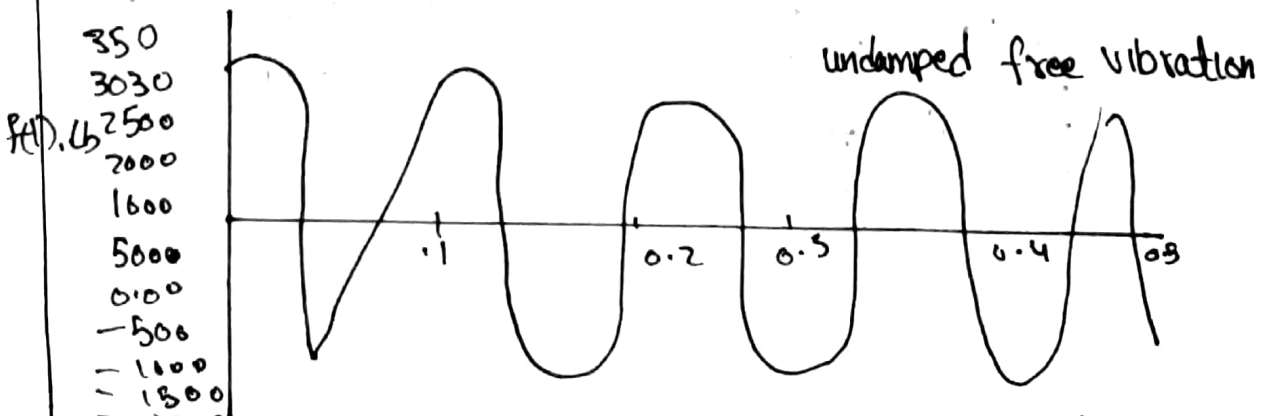
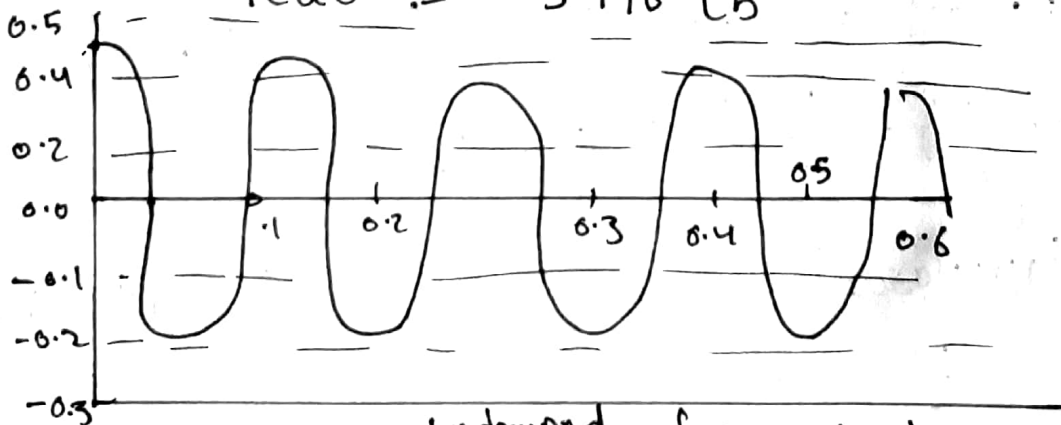
$$u_0 = \frac{1}{24} \text{ ft}$$

Amplitude of equivalent static force

$f_{s0}$

$$K u_0 = 90625 \times \frac{1}{24} = 3776 \text{ lb}$$

$$\underline{K u_0} = 3776 \text{ lb}$$



Q2 Ans

Given data

as given in question statement we will use given data from Question 1

Required

- i) develop and solve the equation of motion for vibration resulting at free end
- ii) develop eq showing variation in equivalent static force with time
- iii) Draw graph to show variation of displacement with time & variation of equivalent force with time

Solution:-

Damping ratio for RCC  
 $M_{mm} = 0.80\%$ , Average = 1.3%

we will consider 1.3%

E.o.m for damped free vibration

$$Ku + Cu + m\ddot{u} = 0 \quad \text{--- A}$$

from question 1

$$K = 90,625 \text{ lb/ft}$$

$$m = 227.8 \text{ lbsec}^2/\text{ft}$$

$$C = \zeta \times 2m \omega_n$$

$$= 2 \times 227.8 \times 20 \times 0.013$$

$$C = 118.45 \text{ lb/ft}$$

Now Put value of  $K$ ,  $m$ , &  $C$   
equation (A)

$$\Rightarrow 90625u + 118.45 \ddot{u} + 227.8 \dot{\ddot{u}} = 0$$

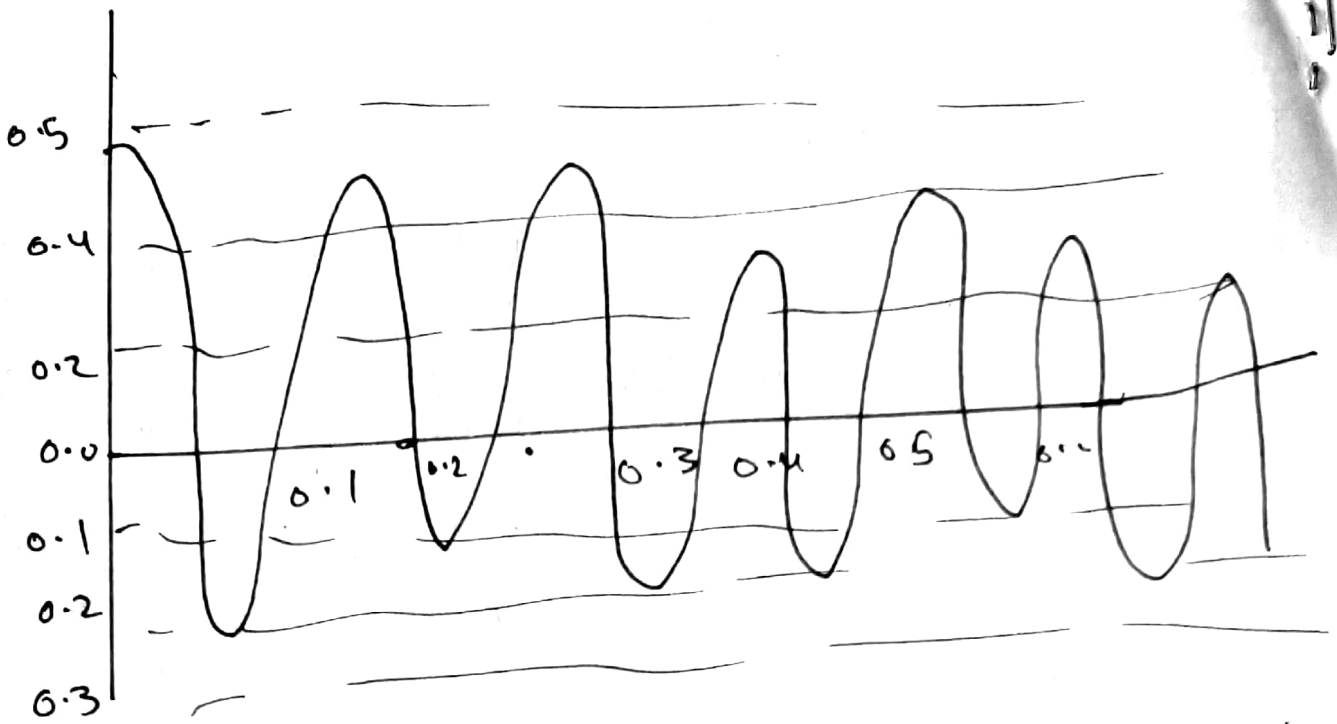
selection to the E.O.M for  
damped free vibration

$$u(t) = e^{-\zeta \omega_n t} \left[ u(0) \cos(\omega_d t) + \frac{1}{\omega_d} \left[ \dot{u}(0) + \zeta \omega_n u(0) \right] \sin(\omega_d t) \right]$$

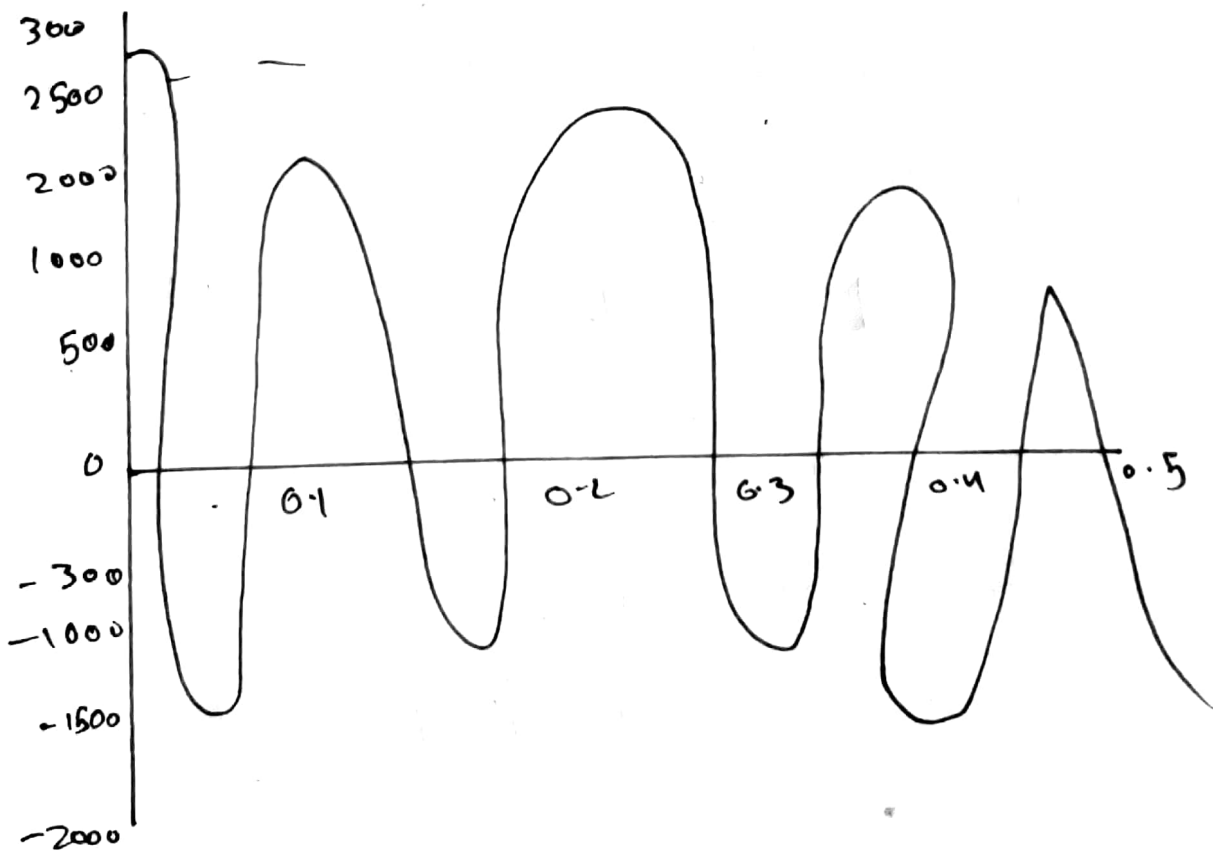
$$u(t) = e^{-0.26t} \left[ 0.041 \cos(20t) + 0.005416 \sin(20t) \right]$$

$$f_s(t) = ku(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.26t} \left[ 3715 \cos(20t) + 49.08 \sin(20t) \right]$$



## Damp free vibration





Q3

Page # 07

Given data

$$\text{force} = F = 60 \text{ kips}$$

$$\text{displacement} = \frac{7337}{1000} = 7.337''$$

no of cycle 7 are complete in 3.57sec

$$\text{Amplitude} = 2.286 \text{ cm} = 0.9''$$

ignore vertical vibration

Required:-

- i) Damping ratio
- ii) Natural Period of undamped vibration
- iii) stiffness of structure
- iv) weight of tank
- v) Damping coefficient
- vi) No of cycle to reduce displacement 0.5''

Solution

$$u_1 = 7.337''$$

$$\text{After } T = 7, u_{i+1} = u_8 = 0.9''$$

$\zeta =$  Damping ratio = ?

$$\zeta = \frac{1}{2\pi n} \ln(7.37/0.9)$$

$$T = \frac{1}{2\pi n} \ln(7.37/0.9)$$

$$\zeta = \frac{2.13}{2\pi T}$$

$$\zeta = 0.488 = 4.88\%$$

$$T_n = ?$$

7 cycle of vibration are  
Completed in 3.57 sec

Now Time required for 1 cycle  
 $= T_0 = 3.57/7$

$$T_0 = 0.51 \text{ sec}$$

Now

$$\omega_0 = \omega_n \sqrt{1 - \zeta^2}$$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$T_0 = \frac{T_n}{\sqrt{1 - \zeta^2}}$$

$$T_n = T_n \times \sqrt{1 - \zeta^2}$$

$$T_n = 0.51 \times \sqrt{1 - (0.0488)^2}$$

$$= 0.5093 = 0.51 \text{ sec}$$

$$k = ?$$

$$= \frac{60 \times \cos 60^\circ}{2} = 1514_m$$

$$k = 18000 \text{ lb/ft}$$

weigh of tank =  $\omega = ?$

$$\omega_n = \frac{k}{m} = \sqrt{\frac{k}{m/g}} = \sqrt{\frac{k \times g}{\omega}}$$

$$\omega_n^2 = \frac{k \times g}{\omega}$$

$$\omega = \frac{k \times g}{\omega_n^2 a}$$

$$\text{also } \omega_n = 2\pi/T_n$$

$$\omega = \frac{kxg}{4\pi^2 T_n^2}$$

$$\omega = kxg \times \frac{T_n^2}{4\pi^2}$$

$$\omega = 18000 \times 32.2 \times \frac{(0.51)^2}{4\pi^2}$$

$$\omega = 3818.64 \text{ lb} = 3.811 \text{ k}$$

Now

$$C = ?$$

It is known that

$$s = \frac{C}{2m\omega_n}$$

$$C = S \times 2m\omega_n$$

$$C = S \times 2m \left( \frac{2\pi}{T_n} \right) = 0.0488 \times 2 \times \left( \frac{3818.6}{31.2} \right) \left( \frac{2\pi}{0.51} \right)$$

$$C = 142.59 \text{ lb}_{\text{sec}} / \text{ft}$$

No we find nos of cycle to reduce displacement amplitude from 7.75 to 0.5"  $J = ?$

$$J = \frac{1}{2\pi \times 0.0488} \ln \left[ \frac{7.337}{0.5} \right]$$

$$J = 8.1 \text{ or } 8 \text{ cycle}$$