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Subject = Differential Equation
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Q NO:1 Solve The following objective types Question.

(i) the order of matric A is m×p and the order of B is p×n then order of matrix AB is?

Sol:

order of A = m×p
Order of B = p×n
So order of A×B = m×n

(ii) the number of non-zero rows in an Echelon form?

Sol: The number of non-zero rows in an Echelon form is called rank of the matrix

for example

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since A is an row-reduced form. Since it can three non-zero rows. it row rank is three ~~non-zero~~ ro.

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(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a Singular matrix

Then: $a = ?$

Sol:

We know that for Singular matric

$|B| = 0$

so

$|B| = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

$1 \times a - 2 \times 4 = 0$

$a - 8 = 0$

$\boxed{a = 8}$ Ans

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:

$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

take modulus of a matrix A

$|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$|A| = 2i \times (-i) - i \times i$

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$$|A| = -2i^2 - i^2$$

$$(\because i^2 = -1)$$

$$|A| = -2(-1) - (-1)$$

$$|A| = 2 + 1$$

$$\boxed{|A| = 3} \text{ — Ans}$$

(V) the matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

Sol:

$$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

Here in matrix A diagonal element is same i.e 9 so it is scaler matrix

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(vi) Solution of $dy/dx + 2xy = y$ = ?

Sol: $dy/dx = y - 2xy$

$dy/dx = y(1-2x) dx$

$dy/y = (1-2x) dx$

$\int dy/y = \int (1-2x) dx$

$\ln y = x - 2x^2/2 + c_1$

$\ln y = (x - x^2) + c_1$

$e^{\ln y} = e^{(x-x^2)+c_1}$

$y = e^{x-x^2} . e^{c_1}$

$\boxed{y = Ce^{x-x^2}}$

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Pg ~~#5~~

(vii) the order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Sol:

Taking square on both sides

$$\left(\left(\frac{dy}{dx}\right)^3\right)^2 = \left(\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2}\right)^2$$

$$\left(\frac{dy}{dx}\right)^6 = 1 + \left(\frac{dy}{dx}\right)^2$$

Degree = 6
order = 1

(viii) the order and degree of differential equation
$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is?

Sol: 2 but the degree is undefined b/c the unknown function y is an argument of transcendental function

(X) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ is ?

Solution:- Expand w.r.t column number 1st

$$1\begin{vmatrix} b & b^2 \\ 0 & c \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$1(bc^2 - cb^2) - 1(ac^2 - ac) + 1(ab^2 - a^2b)$

$(bc^2 - cb^2) - (ac^2 - ac) + (ab^2 - a^2b)$

$bc(a-b) - ac(c-a) - ab(b-a)$

$bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b$

$(c-b)\{bc - ac - ab - a^2\}$

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QNO:2 Express the determinant

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

as the product of factor which are linear in a, b, c

Sol:

$$A = \begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix} = Adj(A) = \begin{bmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{bmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_3A_3 \text{ - - - - } (1)$$

Now

$$A_{11}(-1)^{1+1} M_{11} = (-1)^2 \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$A_{12} = (-1)^{1+1} M_{12} = (-1)^3 \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

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$A_{12} = -\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$

$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$

$= a\left(b^2c^3 - c^2b^3\right) - b\left(a^2c^3 - c^2a^3\right) + c\left(a^2b^3 - b^2a\right)$

$= ab^2c^3 - ac^2b^3 - ba^2c^3 + bc^2a^3 + ca^2b^3 - cb^2a^3$

$(x^2 + 3y^2)\,dy - 2xy\,dy = 0$ at $x = 2, y = 6$

$Mdx + Ndy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = 6y, \quad \frac{\partial N}{\partial x} = -2y$

$\frac{M_y - N_x}{N} = \frac{6y - (-2y)}{-2xy} = \frac{6y + 2y}{-2xy} = \frac{8y}{2xy}$

$= -4/x$

$IF = e^{-\int 4/x\,dx} = e^{-4\int 1/x\,dx} = e^{-4\ln x} = e^{\ln x^{-4}}$

$x^{-4}(x^2 + 3y^2)\,dx - \frac{2xy}{-x^4}\,dy = 6$

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$$\frac{\partial M}{\partial y} = \frac{6y}{x^4} \quad \frac{\partial N}{\partial x} = 6y/x^4$$

$$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

General Solution

$$\int \frac{6y \, dx}{x^2} + \int 0 \, dy = C$$

$$6y/x = C$$

$$-6y = Cx$$

Now at $x = 2, y = 6$

$$-6(6) = C(2)$$

$$-12 = 2C$$

$$C = -6$$

$$-6y/x = -6$$

$$-6y = -6x$$

$$-6y + 6x = 0$$

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Pg # 10

find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol.: $\det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}\right) = 0$$

$$\begin{bmatrix} \lambda-2 & -1 & -1 & 0 \\ -1 & \lambda-3 & -1 & -1 \\ -1 & -1 & -\lambda-3 & -1 \\ 0 & -1 & -1 & \lambda-2 \end{bmatrix}$$

$$\begin{vmatrix} \lambda-3 & -1 & -1 \\ -1 & \lambda-3 & -1 \\ -1 & -1 & \lambda-2 \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & \lambda-3 & -1 \\ 0 & -1 & \lambda-2 \end{vmatrix} + (-1)\begin{vmatrix} -1 & \lambda-3 & -1 \\ -1 & -1 & -1 \\ 0 & -1 & \lambda-2 \end{vmatrix} 0 = 0$$

$x^2 + y^2 = x^3 c$ —— (A)   so putting value of $x = 2$ and $y = 6$ in eq (A)

$$(2)^2 + (6)^2 = (2)^3 c$$
$$4 + 36 = 8c$$
$$40 = 8c$$
$$\boxed{c = 5}$$

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Putting the value of $C=5$ in eq (A)

$$x^2+y^2 = 5x^2$$

$$x^3 = \frac{x^2+y^2}{5}$$ general solution

QNO3:- the rate of charge in the form of differential equation is given by $(x^2+3y^2)dx - 2xy\,dy = 0$ find the general solution at $x=2$ and $y=6$.

Sol:- $(x^2+3y^2)dx - 2xy\,dy$

$$dy/dx = \frac{x^2+3y^2}{2xy} = \frac{1+3(y/x)^2}{2(y/x)}$$

its Homogeneous

put $y = vx$ $= y/x = v$ Diff w.r.t x

$$dy/dx = v + x\,dv/dx$$

$$v + x\,dv/dx = \frac{1+3v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{1+3v^2}{2v} - v$$

$$\frac{x\,dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$\frac{x\,dv}{dx} = \frac{1+v^2}{2v}$$

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Q# 12

$$\frac{2v}{1+v^2}\, dv = \frac{dx}{x}$$

$$\int \frac{2v}{1+v^2}\, dv = dx/x$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$1+v^2 = c$$

$$1+y^2/x^2 = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

Now put $x=2$, $y=6$ we get

$$\frac{4+36}{4} = 2c$$

$$40/4 = 2c$$

$$10 = 2c$$

$$\boxed{c = 5}$$

thus the general solution is given by

$$\frac{x^2+y^2}{x^2} = 5x$$

$$\boxed{x^2+y^2 = 5x^3}$$

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