



**Department of Electrical Engineering**

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**13122**

**Final Examination**

**Electro Magnetic Field Theory**

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**Question 1. (A)**

Determine the magnetic field at the center of the semicircular piece of wire with radius 0.20m. The current carried by the semicircular of wire is 150A.

**Solution:**

The radius of the semicircular piece of wire = 0.20 m

Current carried by the semicircular piece of wire = 150 A

Magnetic field is given as:  $B = \frac{\mu_0 NI}{2a}$

The differential form of Biot-Savart law is given as:

$$dB = \frac{\mu_0 I}{4\pi r^2} dl \sin\theta$$

$$B = \frac{\mu_0 I}{4\pi r^2} \int dl \sin\theta$$

$$= \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi r^2} 2\pi r$$

$$= \frac{\mu_0 I}{2r}$$

$$= \frac{4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} (150\text{A})}{2(0.20\text{m})}$$

$$= 2.4 \times 10^{-4} \text{ T}$$

**Question 1. (B)**

A circular coil of radius  $5 \times 10^{-2}$  m and with 40 turns is carrying a current of 0.25 A. Determine the magnetic field of the circular coil at the center.

**Solution:**

The radius of the circular coil =  $5 \times 10^{-2} \text{m}$

Number of turns of the circular coil = 40

Current carried by the circular coil = 0.25 A

Magnetic field is given as:  $B = \frac{\mu_0 NI}{2a}$

$$= \frac{4\pi \times 10^{-7} \text{T.m/A} (40) (0.25 \text{A})}{2.50 \times 10^{-2} \text{m}}$$

$$= 1.2 \times 10^{-4} \text{ T}$$

**Question 2. (A)**

Compute the magnetic field of a long straight wire that has a circular loop with a radius of 0.05m. 2amp is the reading of the current flowing through this closed loop.

**Solution:**

$$R = 0.05 \text{m}$$

$$I = 2 \text{amp}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{N/A}^2$$

Ampere's law formula is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

In the case of long straight wire

$$\oint d\vec{l} = 2\pi R$$

$$= 2 \times 3.14 \times 0.05 \text{m}$$

$$= 0.314$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B \cdot 2\pi R = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R} = \frac{4\pi \times 10^{-7} \times 2}{0.314}$$

$$B = 8 \times 10^{-6} \text{ T}$$

### Question 2. (B)

Within the cylinder  $\rho = 2$ ,  $0 < z < 1$ , the potential is given by  $V = 100 + 50\rho + 150\rho \sin\phi$ .

**Solution.**

a) Find  $V$ ,  $E$ ,  $D$ , and  $\rho_v$  at  $P(1, 60^\circ, 0.5)$  in free space: First, substituting the given point, we find  $V_P = 279.9 \text{ V}$ . Then,

$$E = -\nabla V = -\left(\frac{\partial V}{\partial \rho}\right)\hat{\rho} - \left(\frac{1}{\rho}\right)\left(\frac{\partial V}{\partial \phi}\right)\hat{\phi}$$

$$= -[50 + 150 \sin\phi]\hat{\rho} - [150 \cos\phi]\hat{\phi}$$

Evaluate the above at  $P$  to find  $E_P = -179.9\hat{\rho} - 75.0\hat{\phi} \text{ V/m}$

Now  $D = \epsilon_0 E$ , so

$$D_P = -1.59\hat{\rho} - 0.664\hat{\phi} \text{ nC/m}^2$$

$$\rho_v = \nabla \cdot D = \left(\frac{1}{\rho}\right)\left(\frac{d}{d\rho}\right)(\rho D_\rho) + \left(\frac{1}{\rho}\right)\left(\frac{\partial D_\phi}{\partial \phi}\right)$$

$$= \left[-\frac{1}{\rho}(50 + 150 \sin\phi) + \left(\frac{1}{\rho}\right)150 \cos\phi\right] \epsilon_0 = -50/\rho \epsilon_0 \text{ C}$$

At  $P$ , this is

$$\rho_{vP} = -443 \text{ pC/m}^3$$

b) How much charge lies within the cylinder? We will integrate  $\rho_v$  over the volume to obtain:

$$Q = \int_V \rho_v dV = \int_0^1 \int_0^{2\pi} \int_0^2 (-50/\rho) \rho d\rho d\phi dz$$

$$= -2\pi(50)\epsilon_0(2)$$

$$= -5.56 \text{ nC}$$

### QUESTION 3.

Given the time-varying magnetic field,  $B = (0.5ax + 0.6ay - 0.3az)\cos 5000t \text{ T}$ , and a square filamentary loop with its corners at  $(2, 3, 0)$ ,  $(2, -3, 0)$ ,  $(-2, 3, 0)$ , and  $(-2, -3, 0)$ , find the time-varying current flowing in the general  $\hat{a}_\phi$  direction if the total loop resistance is  $400 \text{ k}$ .

**Solution:**

As we know that;

$$\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -d\Phi / dt = -d / dt \int_{\text{Loop area}} \mathbf{B} \cdot \mathbf{a}_z \, da = d / dt (0.3)(4)(6)\cos 5000t$$

Where the loop normal is chosen as positive  $\mathbf{a}_z$ , so that the path integral for  $\mathbf{E}$  is taken around the positive  $\mathbf{a}_\phi$  direction. Taking the derivative, we find

$$\text{emf} = -7.2(5000)\sin 5000t$$

so that

$$I = \text{emf} / R$$

$$= -36000\sin 5000t / 400 \times 10^3$$

$$= -90\sin 5000t \text{ mA}$$