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BS (SE-8) Section: A

Submitted To:

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Linear Algebra

Question No 1:-

Sol: Let

$[A - \lambda I]$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -17 & 8 \\ 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} -\lambda & 1 \\ -17 & 8-\lambda \end{vmatrix} -1 \begin{vmatrix} 0 & 1 \\ 4 & 8-\lambda \end{vmatrix} = 0$$

$$-\lambda \left[-\lambda(8-\lambda) + 17 \right] + 4 = 0$$

$$-\lambda \left[-8\lambda + \lambda^2 + 17 \right] + 4 = 0$$

$$-\lambda^3 + 8\lambda^2 - 17\lambda + 4 = 0$$

$$\lambda^3 + 8\lambda^2 + 17\lambda - 4 = 0$$

$\lambda = 4$ is a root.

$$
\begin{array}{c|cccc}
4 & 1 & -8 & 17 & -4 \\
 & \downarrow & 4 & -16 & 4 \\
\hline
 & 1 & -4 & 1 & \boxed{0}
\end{array}
$$

$$t^2 - 4t + 1 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$t = 2 \pm \sqrt{3}$$

Q2)

Sol: A can be diagonalized if there exists an invertible matrix P and diognal matrix D such that $A = PDP^{-1}$

Here $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

$A - \lambda = 0$

$$\begin{vmatrix} (-\lambda) & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & (3-\lambda) \end{vmatrix} = 0$$

$= (-\lambda)((2-\lambda)\times(3-\lambda) - 1\times 0) - 0(1\times(3-\lambda) - 1\times 1) + (-2)(1\times 0) - (2-\lambda)\times 1 = 0$

$= (-\lambda)(((6-5\lambda+\lambda^2) - 0) - 0((3-\lambda)-1) - (2-\lambda) = 0$

$= (-\lambda)(6-5\lambda+\lambda^2) - 0(2-\lambda) - 2(-2+\lambda) = 0$

$= (-6\lambda+5\lambda^2-\lambda^3) - 0 - (-4+2\lambda) = 0$

$= (-\lambda^3+5\lambda^2-8\lambda+4) = 0$

$= -(\lambda-1)(\lambda-2)(\lambda-2)=0$

$= (\lambda-1)=0$ or $(\lambda-2)=0$ or $\lambda-2=0$

The eigenvalues of the matrix A is given by $\lambda = 1, 2$.

eigen vector for $\lambda = 2$

$$V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad V_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

The eigenvector compose the columns of matrix P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The diagnol matrix D is composed of eigen values.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now find P^{-1}

$$|P| = \begin{vmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -2 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -2 \times (1 \times 1 - 0 \times 0) + 0 \times (1 \times 1 - 0 \times 1) - 1 \times (1 \times 0 - 1 \times 1)$$

$$= -2 \times (1 + 0) + 0 \times (1 + 0) - 1 \times (0 - 1)$$

$$= -2 \times 1 + 0 \times 1 - 1 \times (-1)$$

$$= -2 + 0 + 1$$

$$= -1$$

Q3)

Sol:

Here $A = (1,-2,3), B = (5,6,-1),$
$C = (3,2,1)$

The vectors A, B, C are linearly dependent, If their determinant is Zero. ie $|D| = 0$

$$|D| = \begin{vmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (6 \times 1 - (-1) \times 2) + 2 \times (5 \times 1 - (-1) \times 3 +$$
$$3 \times (5 + 2 - 6 \times 3)$$

$$= 1 \times 8 + 2 \times (8) + 3 \times (-8)$$
$$= 8 + 16 - 24$$
$$= 0$$

Since $|D| = 0$ So Vectors A, B, C are linearly independent.

Answer No 4:

Part A):

Definition: A *vector space* is a set V on which two operations + and · are defined, called *vector addition* and *scalar multiplication.*

The operation + (vector addition) must satisfy the following conditions:

Closure: If **u** and **v** are any vectors in V, then the sum $\mathbf{u} + \mathbf{v}$ belongs to V.

(1) *Commutative law:* For all vectors **u** and **v** in V, $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(2) *Associative law:* For all vectors **u**, **v**, **w** in V, $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

(3) *Additive identity:* The set V contains an *additive identity* element, denoted by **0**, such that for any vector **v** in V, $\mathbf{0} + \mathbf{v} = \mathbf{v}$ and $\mathbf{v} + \mathbf{0} = \mathbf{v}$.

(4) *Additive inverses:* For each vector **v** in V, the equations $\mathbf{v} + \mathbf{x} = \mathbf{0}$ and $\mathbf{x} + \mathbf{v} = \mathbf{0}$ have a solution **x** in V, called an *additive inverse* of **v**, and denoted by - **v**.

The operation · (scalar multiplication) is defined between real numbers (or scalars) and vectors, and must satisfy the following conditions:

Closure: If **v** in any vector in V, and c is any real number, then the product $c \cdot \mathbf{v}$ belongs to V.

(5) *Distributive law:* For all real numbers c and all vectors **u**, **v** in V, $c \cdot (\mathbf{u} + \mathbf{v}) = c \cdot \mathbf{u} + c \cdot \mathbf{v}$

(6) *Distributive law*: For all real numbers c, d and all vectors **v** in V, $(c+d) \cdot \mathbf{v} = c \cdot \mathbf{v} + d \cdot \mathbf{v}$

(7) *Associative law*: For all real numbers c,d and all vectors **v** in V, $c \cdot (d \cdot \mathbf{v}) = (cd) \cdot \mathbf{v}$

(8) *Unitary law*: For all vectors **v** in V, $1 \cdot \mathbf{v} = \mathbf{v}$

Q4)

The set $P_n(x)$ of all polynomials over R in variable x of degree $\leq n$ forms a vector space over R.

If $f_n(x) = a_0 + a_1 x + \ldots\ldots\ldots\ldots + a_n x^n$

and $g_n(x) = b_0 + b_1 x + \ldots\ldots\ldots + b_n x^n$, $a_i, b_i \in R$

Then $f_n(x) + g_n(x) = (a_0 + b_0) + (a_1 + b_1)x + \ldots\ldots$
$\ldots + (a_n + b_n)x^n$

The associative additive property is induced from the additive associative property of R.

The zero polynomial $f_0(x) = 0$ of degree 0 acts as the additive identity of $P(x)$

and

$-f_0(x) = -a_0 + (-a_1)x + \ldots + (-a_n)x^n$ is additive inverse of $f_n(x)$

Commutative property follows from the commutative property of R. Hence $P_n(x)$ is an additive abelian group.

The scalar multiplication of $a \in R$ by $f_n(x)$ is defined by

$$a.f_n(x) = aa_0 + (aa_1)x + (aa_2)x_2 \dots \dots + (aa_n)x^n \in P_n(x).$$

It observes properties of scalar multiplication which can easily be verified. So that $P_n(x)$ forms a vector space over R.

Q4

b)

Let f be a field. A non-empty set V together with two binary operations (+) and (−) forms the algebric structure of a vector space over the field F, if,

a) V forms an additive (+ve) abelion group

b) The scalar multiplication (.) as a function from $F \times V$ into V Observes the following properties;

i) $\forall \alpha \in F, x, y \in V,\ a.(x+y) = ax + ay$

ii) $\forall a, B \in F, (a+B).x = ax + Bx, \forall x \in V$

iii) $\forall a, B \in F, \forall x \in V;\ a(Bx) = (aB)x$

iv) For e_F, the identity of f $e_F.x = x$, $\forall x \in V$.