Day: M T W T F S Date: 21 / 08/ 2020

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Subject # Applied Calculus.
Section # "A"
Semister # 1st
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Exam # Mid Summar Exam.

Q 2:- $Y(x) = x^2 + \sin x$

Sol:- By Maclaurin's Series expansion, We have,

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots$$

Now, $f(x) = Y(x) = x^2 + \sin x$

$f'(x) = 2x + \cos x$

$f''(x) = x - \sin x$

$f'''(x) = -\cos x$

Thus,

$f(0) = (0)^2 + \sin(0) = 0$

$f'(0) = 2(0) + \cos(0) = 1$

$f''(0) = 0 - \sin(0) = 0$

$f'''(0) = -\cos(0) = -1$

Hence by Maclaurin's Expansion,

$$f(x) = Y(x) = 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(-1)}{3!}$$

$$= 0 + x + 0 - \frac{x^3}{3!} + \cdots\cdots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

is the required Maclaurin's Expansion.....

Q. Find y by logarithmic differentiation.

$$y = x^3(1+x)^9 e^{6x}$$

Sol:

$$y = x^3 (1+x)^9 e^{6x}$$

taking Ln on b/s

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$ ①

using log property $\because \ln e^a = a\cancel{\log e}$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x.$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 3 \frac{1}{x} + 9 \frac{1}{x+1} + 6x$$

$$\frac{dy}{dx} = 3 \frac{y}{x} + 9 \frac{y}{x+1} + 6xy$$

$$\frac{dy}{dx} = 3 \frac{x^3 (x+1)^9 e^{6x}}{x} + 9 \frac{x^3 (x+1)^{\cancel{9}\,8} e^{6x}}{\cancel{x+1}} + 6x \cdot x^3 (x+1)^9 e^{6x}$$

$$= \boxed{3x^2(x+1)^9 e^{6x} + 9x^3(x+1)^8 e^{6x} + 6x^4 (x+1)^9 e^{6x}}$$

Q3:- $1 + xy = x^2 + y^2$

Sol:- $\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$

$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$

$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x + 2y\frac{dy}{dx}$

$= \frac{x\,dy}{dx} + y = 2x + 2y\frac{dy}{dx}$

$= \frac{x\,dy}{dx} - \frac{2y\,dy}{dx} = 2x - y$

$= \frac{dy}{dx}(x-2y) = 2x - y$

$\frac{dy}{dx} = \frac{2x-y}{x-2y}$ ——— (1)

Day: M T W T F S P3 Date: 21/08/2020

Q3:- (1) $y' = \frac{dy}{dx} = \frac{2x-y}{x-2y}$

$= \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$ Quatent Rules

$= \frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$

$= y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$

$= y'' = \frac{(2x - xy' - 4y + 2yy') - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$

$= y' = \frac{4xy' - 3y - xy'}{(x-2y)^2}$

$= y'' = \frac{4x\left(\frac{2x-y}{x-2y}\right) - 3y - x\left[\frac{2x-y}{x-2y}\right]}{(x-2y)^2}$

$= y'' = \frac{4x(2x-y) - 3y(x-2y) - x(2x-y)}{(x-2y)(x-2y)^2}$

$= y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$

Checked By: .... Parents .... Excellent ☐ Good ☐ BABAR PAPER PRODUCTS

$2t+3$

Q1) The function $g(t)$ is defined by

$g(t) = 0 \quad t < 0$

$t^2 \quad ~~1|t|10~~ \; 0 \le t \le 3$

$2t+3 \quad 3 < t \le 4$

$12 \quad t > 4$

a) State any point of discontinuity
b) Find, if they exist
i) $\lim_{t \to 3} g$

Sol:-

a) To check possibility of the discontinity of the function is at $t=0$ & 4

→ First at $t=0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

Apply limits

$= 1 + 0^2 + \xi 2(0)$

$= 1$

For L.H.L

$\lim_{h\to 0} g(1-h) = 2t+3$

$= \lim_{h\to 0} 2(1-h)+3.$

$= \lim_{h\to 0} 2-2h+3$

Apply limit

$= 2-2(0)+3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

→ Now at $t = 4$

$g(4) = 2(4)+3$

$= 8+3$

$= \underline{11}$

For R.H.L

L.H.L $\lim_{h\to 3} g(1-h) = \lim_{h\to 3}$

$= \lim_{h\to 3} 2(1-h)+3$

(2)

(3) (1)

$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$

$= \lim_{h\to 0} 2+2h+3$

Apply limits

$= 2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim_{h\to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t=4$

b) Find, if they exist

i) $\lim_{t\to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h\to 3} g(1+h) = \lim_{h\to 3} (1+h)^2$

$= \lim_{h\to 3} 1+h^2+2h$

Apply limits

$= 1+3^2+2(3) \Rightarrow 16$

L.H.L $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)