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Section	C
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Subject	Intr. Earthquack and Structure dynamic
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Q = No: 01

Solution: Given data:

* A beam is placed in a downward direction $= \frac{1}{2}$ in
* Ignore the Self weight of beam as well as clamping effect.
* $E = 29000\,ksi$
* $I = 150\,in^4$
* δ_{st} : Deflection due to 7685 lb Static load

Required data:

* Natural time period of system = ?
* Develop and Solve equation of motion for vibration

Solution: As we ~~kh~~ know that

The general EOM for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undampeel $(c=0)$ underging free vibration $(p(t)=0$

Hence general EOM become

$$ku + m\ddot{u} = 0 \longrightarrow (1)$$

$$k = \frac{2EI}{L^3} = \frac{2 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^2}$$

$$\boxed{k = 7.55 k/in}$$

In order to eliminate the chance of mistake during Calculation, it is more appropriate to use fundomental units like lb, ftsec or kg, m, sec

→ $k = 7.55\ k/in = 90625\ lb/ft$

→ $m = \frac{7685\ lbsec^2}{32.2 ft} = 238.66$ Slug

$$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{238.66}} = 19.48\ rad/sec$$

$$T_n = \frac{2\pi}{W_n} = \frac{2\pi}{19.48} = 0.322\ sec$$

Substituting the Corresponding values in eq(1)

$$ku + m\ddot{u} = 0$$

$$\cancel{6025}\ 90625u + 238.66\ddot{u} = 0$$

Where "k" in lb/ft and "m" in lb sec/ft²

General Solution to the EOM for undamped free vibration is,

P-T-O

By Substn.

③

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$u(0) = \frac{1}{2}''$, $\frac{1}{2\times12} = \frac{1}{24}$ and $\dot{u}(0) = 0$

$$u(t) = \frac{1}{24} \times \cos(19.48t) + 0$$

equivalent Static force at any time "t" is =

$$f_s(t) = \overset{K.u(t)}{\cancel{3776\cos(19.48t)}} = 3776\times\cos(19.48t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0} \quad \dot{u}(0) = 0$$

$$\boxed{u_0 = \frac{1}{24}\,ft}$$

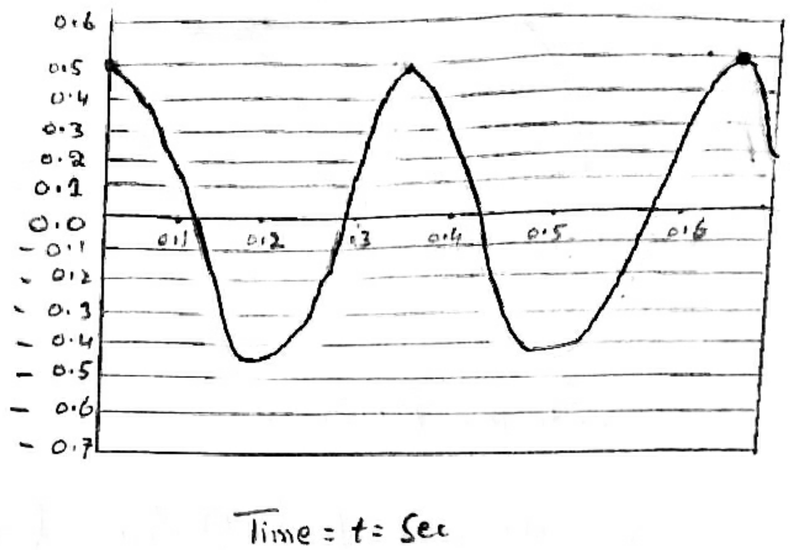
Amplitude of equivalent Static force, f_{s0}

$$Ku_0 = 90625 \times \frac{1}{24} = 3776\,lb$$

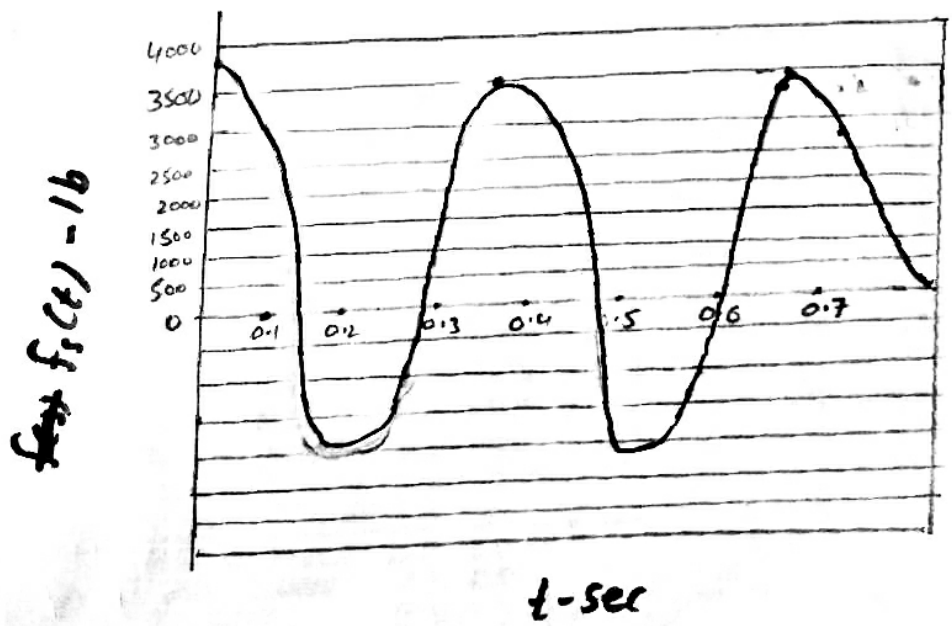
$$\boxed{Ku_0 = 3776\,lb}$$

Graphs:

a: Showing variation of displacement with time:



(b) Showing variation equivalent Static force with time



Q = No = 02

(4)

Given data

→ ξ (Damping ratios of Reinforced Concrete with Considerable Cracking = 3-5% = 3%.

→ $K = 90625 \text{ lb/ft}$

→ $m = 238.66 \text{ Slug} = 238.66 \frac{\text{lb sec}^2}{\text{ft}}$

→ $\omega_n = 19.48 \text{ rad/sec}$

→ $u(0) = \frac{1}{24}$

Required:

→ Develop and Solve the equation showing Variation in equivalent static force with time.

→ Draw graph to show the variation of displacement with time and the Variation of equivalent static force with time.

Solution: EOM for damped free vibration is:

$$Ku + C\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

~~$C = \xi \times 2m\omega_n = 0.003 \times 2 \times 238.66$~~ $\quad \xi = 3\% = \cancel{0.003}\ 0.03$

$$c = \xi \times 2m\omega_n = \cancel{0.003}\ 0.03 \times 2 \times 238.66 \times 19.48 = 278.9 \text{ lb.sec/ft}$$

$$\boxed{c = 278.9 \text{ lb s/ft}}$$

By Substituting values of k, c and m in (1) we get

$$90625u + 278.9\dot{u} + 238.66\ddot{u} = 0$$

Solution of EOM for damped free vibration is:

$u(t) = e^{-\xi \omega_n t} [u(0)\cos(\omega_D t) + \frac{1}{\omega_D}[\dot{u}(0) + u(0)\xi \omega_n] \cdot \sin(\omega_D t)$

$\omega_D = 19.48 \, rad/sec$

$u(t) = e^{-0.03\times 19.48t} [\frac{1}{24}\times\cos(19.48t)] + [\frac{1}{19.48}\times[0 + \frac{1}{24}\times 0.03\times 19.48]\sin(19.48t)$

$u(t) = e^{-0.5844t} \times [0.041\times\cos(19.48t) + 0.051(0 + 0.02435)\sin(19.48t)$

$u(t) = e^{-0.5844t} \times [0.041\cos(19.48t) + 0.0012\sin(19.48t)$

$f_s(t) = K.u(t) = 90625\times u(t) \rightarrow (2)$

putto u(t) in eq (2)

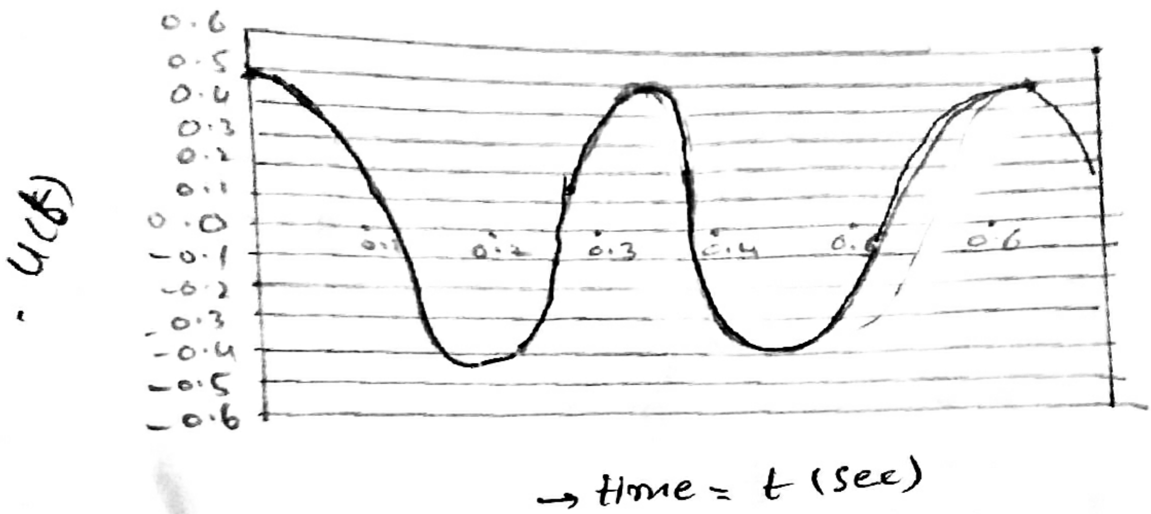
$f_s(t) = 90625(e^{-0.5844t} \times (0.041\cos(19.48t)) + 90625(0.0012\sin(19.48t)$

$f_s(t) = e^{-0.5844t} \times [3776\cos(19.48t)] + [10875\times\sin(19.48t)]$

$f_s(t) = e^{-0.5844t} \times [3776\cos(19.48t)] + [108.75\sin(19.48t)]$

kips — 7.685in

Graph show displacement with time.



(b) Graph show equivalent static force with time.

Q=No=03

Given data

→ Amplitude Cable force = 60 kips

→ Horizental displacement of tank = $\frac{7685}{1000}$ = 7.685 in

→ Cycle = 7

→ Cyclem Completion time = 3.57 sec

→ Amplitude of displacement = 2.286 cm = 0.9 in

Required data: Complete the following

a → Damping ratio

b → Natural perod of undamped vibration

c → Stiffness of Structure

d → Weight of tank

e → Damping Co-efficient

f → Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

→ Displacement of tank, $u_1 = 7.685''$

→ After 7 cycles, i.e After $J = 7$, $u_{J+1} = u_8 = 0.9''$

(a) Damping ratio = ξ = ?

$$J = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{J+1}}\right] = \frac{1}{2\pi\xi} \ln\left[\frac{7.685}{u_{J+1}}\right]$$

$$7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.685}{0.9}\right]$$

$$7 = \frac{1}{2\pi\xi} \times (2.1446)$$

$$\xi = \frac{1}{2\pi\times 7} \times 2.1446$$

$$\xi = 0.0487 = 4.87\%$$

$$\boxed{\xi = 4.87\%}$$

(b) Natural period of undamped vibration = T_n = ?

As the 7 cycles of vibrations are completed in 3.57sec

=> Time required to complete one cycle,

$$T_D = \frac{3.57}{7} = 0.51sec$$

$$\boxed{T_D = 0.51sec}$$

Now $\omega_D = \omega_n \sqrt{(1-\xi^2)}$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{(1-\xi^2)}}$$

$$T_n = T_D \times \sqrt{1 - \xi^2}$$

$$T_n = 0.51 \times \sqrt{1 - (0.0487)^2}$$

$$T_n = 0.51 \times 0.998$$

$$\boxed{T_n = 0.509\,Sec}$$

c) Stiffness of Structure $k = ?$

$$k = \frac{60 \times \cos 60^\circ}{7.685} = 3.90\,k/in$$

$$\boxed{k = 3.90\,k/in = 46800\,lb/ft}$$

(d) Weight of tank, $W = ?$

$$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\frac{W}{g}}} = \sqrt{\frac{k \cdot g}{W}}$$

$$W_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{W_n^2}$$

Also $W_n = \frac{2\pi}{T_n}$

$$W = \frac{k \times g}{\frac{4\pi^2}{T_n^2}}$$

$$W = k g \times \frac{T_n^2}{4\pi^2} = \frac{46800 \times 32.2\,\frac{ft}{sec^2} \times (0.509)^2}{4\pi^2} = 9889\,lb$$

$$\boxed{W = 9.889K}$$

(e) Damping co-efficient, c=?

As we know that

$$\zeta = \frac{C}{2mW_n}$$

$$\Rightarrow C = \zeta \times 2mW_n = \zeta \times 2m \times (2\pi/T_n)$$

$$= \frac{0.0487 \times 4 \times \pi \times \left(\frac{9889}{32.2}\right)}{0.51} = \frac{0.0487 \times 4 \times \pi \times 307.11}{0.51}$$

$$\boxed{C = 368.52 \text{ lb.sec/ft}}$$

f Number of cycle to reduce the displacement amplitude to 0.5"

J=?

$$J = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

$$J = \frac{1}{2\pi \times 0.0487} \times \ln\left[\frac{7.685}{0.5}\right]$$

J = 8.92 or 9 cycles

$$\boxed{J = 8.92 \text{ or } 9 \text{ cycles}}$$

End