NAME :- ABDULLAH JAN

ID :- 16105

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{Civil Engineering}

Q1 The function $g(t)$ is defined by

$$g(t) = \begin{matrix} 0 & t < 0 \\ t^2 & 0 \leq t \leq 3 \\ 2t+3 & 3 < t \leq 4 \\ 12 & t > 4 \end{matrix}$$

Solution

$$g(t) \begin{cases} 0 & ; \ t < 0 \\ t^2 & ; \ 0 < t \leq 4 \\ 2t+3 & ; \ 3 < t \leq 4 \\ 12 & ; \ t > 4 \end{cases}$$

ⓐ we check discontinuity at $x = 4$

Now

i) $f(4) = 2(4) + 3$

$f(4) = 11$

ii) $\lim\limits_{x \to 4^+} f(t) = \lim\limits_{x \to 4} + (12)$

$\lim\limits_{x \to 4^+} f(t) = 12$

and

$\lim\limits_{x \to 4^-} f(t) = \lim\limits_{x \to 4^-} (2(t) + 3)$

$= 2(4) + 3$

$\lim\limits_{x \to 4^-} f(t) = 11$

As

$$\lim_{x \to 4^-} f(t) \neq \lim_{x \to 4^+} f(t)$$

So at $x = 4$ $g(t)$ is discontinuous

or

$x = 4$ is the Point of discontinuity.

(b) Find, if they exist

i $\lim_{t \to 3} g$

Solution

$$\lim_{t \to 3} g(t) = ?$$

$$g(t) = \begin{cases} 0 & ; t < 0 \\ t^2 & ; 0 \leq t \leq 3 \\ 2t+3 ; & 3 < t \leq 4 \\ 12 & ; t > 4 \end{cases}$$

Now Right limit is,

$$\lim_{t \to 3^+} g(t) = \lim_{t \to 3^+} (2t+3)$$

$$= 2(3)+3$$

$$\lim_{t \to 3^+} g(t) = 9$$

Left limit is,

$$\lim_{t \to 3^-} g(t) = \lim_{t \to 3^-} (t^2)$$

$$= (3)^2$$

$$\lim_{t \to 3^-} g(t) = 9$$

$$\therefore \lim_{t \to 3^-} g(t) = 9 = \lim_{t \to 3^+} g(t)$$

Therefore;

$$\lim_{t \to 3} g(t) = 9$$

Q2: Find the Maclaurin's series for

$$y(x) = x^2 + \sin x$$

Solution

The Maclaurin Series is given by,

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \cdots \quad -①$$

$$f(x) = x^2 + \sin x \Rightarrow f(0) = (0)^2 + \sin(0) = 0$$

$$f'(x) = 2x + \cos x \Rightarrow f'(0) = 2(0) + \cos(0) = 1$$

$$f''(x) = 2 - \sin x \Rightarrow f''(0) = 2 - \sin(0) = 2 - 0 = 2$$

$$f'''(x) = 0 - \cos x \Rightarrow f'''(0) = -\cos(0) = -1$$

$$f^{iv}(x) = +\sin x \Rightarrow f^{iv}(0) = \sin(0) = 0$$

Putting all the values in ① ⇒

$$f(x) = 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(1) + \frac{x^4}{4}(0) + \cdots$$

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$$= x + \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3!} + \cdots$$

$$f(x) = x + x^2 - \frac{x^3}{3!}$$

Which is Required Maclaurin's Series.

Q3 i

Find y'' given

$$1 + xy = x^2 + y^2$$

Solution

Diff w.r.t x'

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y(1) = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$(x-2y)\frac{dy}{dx} = (2x-y)$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y} \quad \text{--- (1)}$$

Again diff w.r.t $x \Rightarrow$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

using (1)

$$= \frac{(x-2y)\left(2-\frac{2x-y}{x-2y}\right) - (2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{\cancel{(x-2y)}\dfrac{\left(2(x-2y)-(2x-y)\right)}{\cancel{(x-2y)}} - (2x-y)\dfrac{\left(x-2y-2(2x-y)\right)}{(x-2y)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(\cancel{2x} - 4y - \cancel{2x} + y) - \frac{(2x-y)(x - \cancel{2y} - 4x + \cancel{2y})}{(x-2y)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-3y - \frac{(2x-y)(-3x)}{(x-2y)}}{(x-2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-3y(x-2y) - (2x-y)(-3x)}{(x-2y)^2(x-2y)}$$

$$= \frac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{6(x^2 + y^2 - xy)}{(x-2y)^3}$$

$\because x^2+y^2 = 1+xy$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{6(1+\cancel{xy}-\cancel{xy})}{(x-2y)^3}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{6}{(x-2y)^3} \text{ Ans.}$$

ii Find y' by using logarthmic differentiation $y = x^3(1+x)^9 e^{6x}$

Given that

$$y = x^3(1+x)^9 e^{6x}$$

Taking natural log on both sides,

$$\ln y = \ln\left[x^3(1+x)^9 e^{6x}\right]$$

$\log_a MN = \log_a M + \log_a N$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln(e^{6x})$$

$$\ln y \ 3\ln x + 9\ln(1+x) + 6x\ln e$$

$\because \ln e = 1$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x$$

Diff w.r.t $x \Rightarrow$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(3\ln x + 9\ln(1+x) + 6x\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(3\ln x) + \frac{d}{dx}(9\ln(1+x)) + \frac{d}{dx}(6x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x}\frac{d}{dx}(1+x) + 6$$

$$\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{9}{1+x}(1) + 6\right]$$

$$\frac{dy}{dx} = y\left[\frac{3}{x} + \frac{9}{1+x} + 6\right]$$

$$= y\left[\frac{3(1+x) + 9x + 6x(x+1)}{x(1+x)}\right]$$

$$= y\left[\frac{3 + 3x + 9x + 6x^2 + 6x}{x(1+x)}\right]$$

$$\frac{dy}{dx} = y\left[\frac{6x^2 + 18x + 3}{x(1+x)}\right]$$

$\because y = x^3(1+x)^9 e^{6x}$

$$\frac{dy}{dx} = x^{\cancel{3}\,2}(1+x)^{\cancel{9}\,8} e^{6x}\left[\frac{6x^2+18+3}{\cancel{x}(1+\cancel{x})}\right.$$

$$= x^2(1+x)^8 e^{6x}(6x^2+18x+3)$$

common "3"

$$\frac{dy}{dx} = 3x^2(1+x)^8 e^{6x}(2x^2+6x+1)$$

Ans.